Superconducting black holes

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1. Black hole hair and phase transitions

"Black holes have no hair." Why? Consider four-dimensional Schwarzschild:

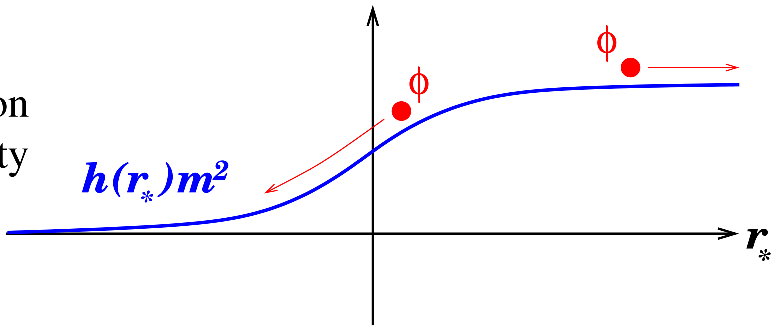
$$ds^2 = -h(r)dt^2 + \frac{dr^2}{h(r)} + r^2 d\Omega_2^2 \qquad h(r) = 1 - \frac{2M}{r} \,. \tag{1}$$

For small perturbations of ϕ, consider only the scalar equation of motion:

$$(\Box - m^2)\phi = 0 \qquad \Longrightarrow \qquad (\partial_t^2 - \partial_{r_*}^2 - h(r_*)m^2)\phi = 0 \tag{2}$$

if we assume $\phi = \phi(t, r)$ and set $dr_* = dr/h$.

Scalar either rolls into horizon ($r_* \to -\infty$) or out to infinity ($r_* \to +\infty$).



Up to some narrow caveats, it's known that no amount of fiddling with multiple scalars will help a black hole "catch" a condensate. See e.g. [Bekenstein 1996].

Gauge fields completely change the story. Consider for example [Gubser 2005]

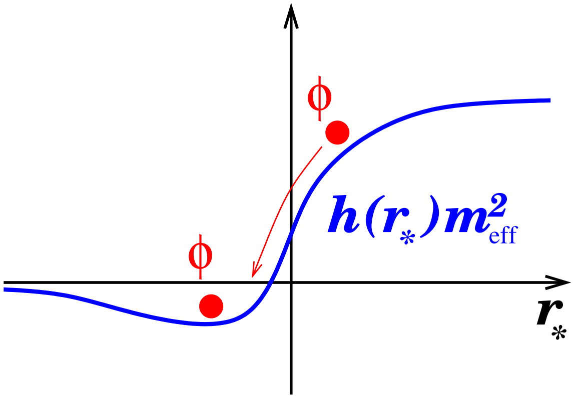
$$\mathcal{L} = \frac{R}{16\pi G_N} - \frac{1}{2}(\partial\phi)^2 - \frac{f(\phi)}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2\phi^2 \qquad f(\phi) = 1 + \ell^2\phi^2$$
$$\Box\phi = \frac{\partial V_{\text{eff}}}{\partial\phi} \qquad V_{\text{eff}}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}f(\phi)F_{\mu\nu}^2 \tag{3}$$

Reissner-Nordstrom has $A_\mu dx^\mu = \Phi dt \qquad \Phi = Q\left(\frac{1}{r} - \frac{1}{r_H}\right)$.

The equation for small fluctuations of ϕ around 0 is

$$\Box\phi = m_{\text{eff}}^2\phi \equiv \frac{\partial V_{\text{eff}}}{\partial\phi} \qquad m_{\text{eff}}^2 = m^2 + \frac{\ell^2}{2}F_{\mu\nu}^2 \tag{4}$$

- Intense electric field tends to make $m_{\text{eff}}^2 < 0$ near the horizon.
- Reissner-Nordstrom solution is then unstable for $T < T_c$.
- Solutions with $\phi \neq 0$ are thermodynamically preferred for $T < T_c$.



I would suggest two broad-brush conclusions:

1. Charged black holes can exhibit second order phase transitions, and the hairy, low-temperature black holes spontaneously break a symmetry:
   - $\mathbf{Z}_2$ for the explicit construction I gave.
   - $U(1)$ if we complexify ϕ and choose $f = f(|\phi|)$, $V = V(|\phi|)$. Weakly gauging such a $U(1)$ with an *additional gauge field* leads to first proposed example of a superconducting black hole—kind of complicated.

2. Stable, hairy black holes in four-dimensional asymptotically flat space seem to be impossible if the matter lagrangian is renormalizable.
   - If non-renormalizable terms in $\mathcal{L}_{\text{matter}}$ are suppressed by a small length ℓ, and there is a mass gap Δ for matter excitations, then hairy black holes can't have r_H much bigger than $r_0 \equiv \frac{\ell}{\Delta\sqrt{G_N}}$.
   - It's strange to see renormalizability play a role in a revised no-hair conjecture; but the conjecture I gave fits the facts to date.

2. Superconducting black holes

In the spirit of [Weinberg 1986], I equate "superconducts" to "spontaneously breaks a $U(1)$ gauge symmetry."

If $m_{\rm eff}^2$ for a complex scalar ψ is negative enough, we'll get $\langle\psi\rangle \neq 0$, breaking the $U(1)$ of its phase.

The simplest possible setup in AdS_4 (really any AdS_{d+1}) is [Gubser 2008b; Hartnoll et al. 2008]

$$\mathcal{L} = \frac{1}{2\kappa^2}\left[R - \frac{1}{4}F_{\mu\nu}^2 - |(\partial_\mu - iqA_\mu)\psi|^2 + \frac{6}{L^2} - m^2|\psi|^2\right] . \tag{5}$$

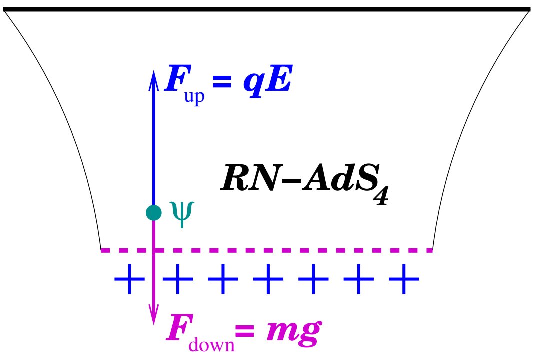
If we assume $A_{(1)} = \Phi dt$ and look at $|\psi|^2$ terms, we see that

$$m_{\rm eff}^2 = m^2 + q^2\Phi^2 g^{tt} . \tag{6}$$

Since $g^{tt} < 0$, we can make $m_{\rm eff}^2$ very negative with very big q. $\Phi \to 0$ at horizon in order for Φdt to be well-behaved, so $m_{\rm eff}^2 \to m^2$ at horizon.

If we did this in flat space, we'd get super-radiant discharge of the black hole's electric charge into scalars. Anti-de Sitter asymptotics prevent that from happening...

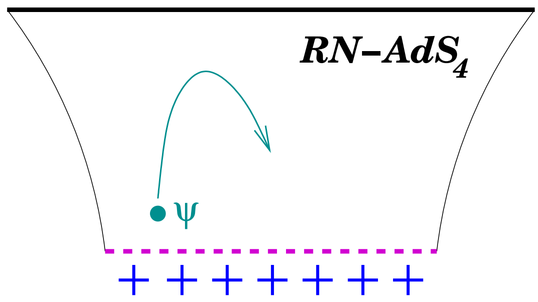
Below some temperature, quanta of ψ are driven upward from horizon: recall $T = g/2\pi$.



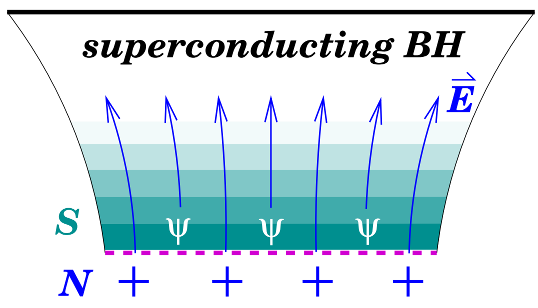
Condensate spontaneously breaks $U(1)$ gauge symmetry, so this is a superconductor: s-wave since ψ is a scalar.

Some fraction of charge remains in "normal" state, behind the horizon.

ψ quanta can never escape from AdS_4, so they fall back toward horizon.



Expected end state has an "atmosphere" of ψ quanta condensed above the horizon.



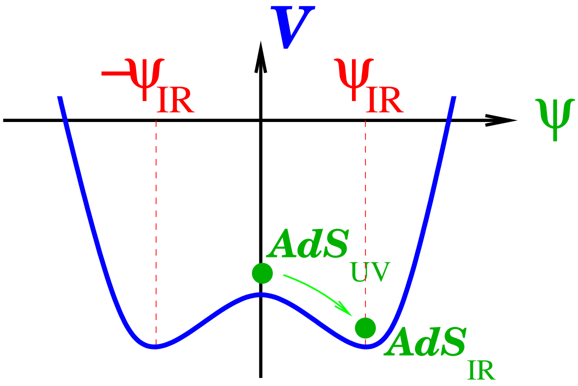
Many questions follow naturally, including:

1. What physics are we describing in the dual CFT? Superconductivity [Gubser 2008b; Hartnoll et al. 2008] and/or superfluidity [Basu et al. 2008; Herzog et al. 2008].
2. What is the zero-temperature limit of superconducting black holes? [Gubser and Rocha 2009; Gubser and Nellore 2009]
3. Can we explicitly embed superconducting black hole solutions in string/M theory? [Denef and Hartnoll 2009]
4. Can we replace ψ with a field with spin? [Gubser 2008c; Roberts and Hartnoll 2008; Gubser and Pufu 2008; Liu et al. 2009; Cubrovic et al. 2009]...
5. What are the interesting probes of superconducting black holes? [Hartnoll et al. 2008; Albash and Johnson 2008; Yarom 2009]... (a few of my favorites)
6. Are there logically distinct mechanisms for superconductivity? [Hartnoll et al. 2008]
7. Additional refs can be found in [Hartnoll 2009; Herzog 2009].

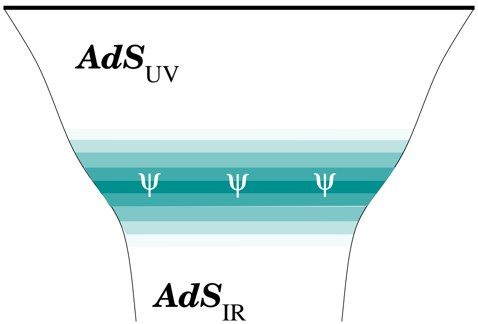
Next let's look at #2, where key concept is *emergent symmetry*. #3 and #4 later.

The most transparent setup in AdS_4 is [Gubser and Rocha 2009]

$$\mathcal{L} = \frac{1}{2\kappa^2}\left[R - \frac{1}{4}F_{\mu\nu}^2 - |(\partial_\mu - iqA_\mu)\psi|^2 - V(|\psi|)\right]$$
$$V(|\psi|) = -\frac{6}{L^2} + m^2|\psi|^2 + \frac{u}{2}|\psi|^4 \qquad m^2 < 0, \ \ u > 0 \tag{7}$$



- A domain wall between $AdS_{\rm UV}$ and $AdS_{\rm IR}$ involving only scalars is a *holographic RG flow*, and describes dynamics of $\mathcal{L}_{\rm CFT} + m_{\rm soft}^{4-\Delta_\psi}\mathcal{O}_\psi$.
- Instead, I want $\mathcal{L}_{\rm CFT}$ undeformed. A scale is set by $U(1)$ charge density ρ in CFT. One finds a *different* domain wall from $AdS_{\rm UV}$ to $AdS_{\rm IR}$.



- $F_{\mu\nu} \to 0$ in $AdS_{\rm IR}$. *All* the charge is carried by the domain wall.

Ansatz for charged domain wall:

$$\begin{aligned} ds^2 &= e^{2A}(-h dt^2 + dx^2 + dy^2) + \frac{dr^2}{h} \\ A_{(1)} &= \Phi(r) dt \qquad\qquad \psi = \psi(r) \end{aligned} \tag{8}$$

Full equations of motion:

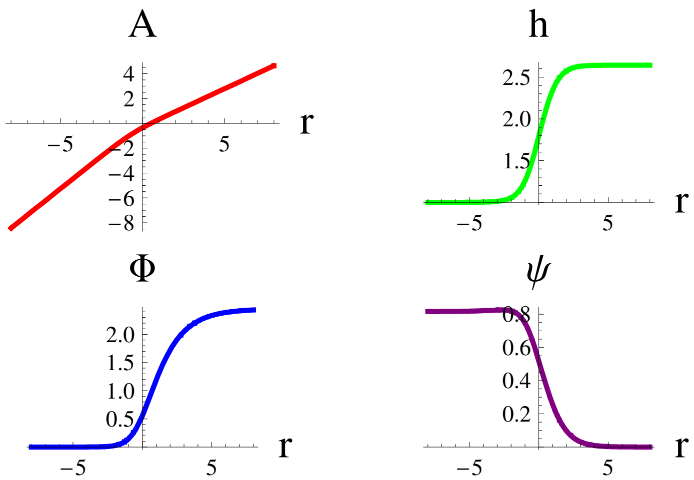
$$\begin{aligned} A'' &= -\frac{1}{2}\psi'^2 - \frac{q^2}{2h^2 e^{2A}}\Phi^2\psi^2 \leq 0 \\ h'' + 3A'h' &= e^{-2A}\Phi'^2 + \frac{2q^2}{h e^{2A}}\Phi^2\psi^2 \geq 0 \\ \Phi'' + A'\Phi' &= \frac{2q^2}{h}\Phi\psi^2 \\ \psi'' + \left(3A' + \frac{h'}{h}\right)\psi' &= \frac{1}{2h}V'(\psi) - \frac{q^2}{h^2 e^{2A}}\Phi^2\psi \,, \end{aligned} \tag{9}$$

- "c-theorem:" $A'_{\rm IR} > A'_{\rm UV}$. Radius of $AdS_{\rm IR}$ is *smaller*. As in [Girardello et al. 1998; Distler and Zamora 1999; Freedman et al. 1999].
- "h-theorem:" $h_{\rm IR} < h_{\rm UV}$. Light travels slower in IR as measured by dx/dt.

Non-zero Φ means there is some finite density $\langle J_0 \rangle$ of a dual charge density.

We prescribe $\psi \sim e^{-\Delta_\psi r}$, dual to some VEV $\langle \mathcal{O}_\psi \rangle$, with *no deformation* of $\mathcal{L}_{\rm CFT}$.

Recovering AdS_4 in the IR (constant ψ, constant h, linear A) means you have *emergent conformal symmetry* in the IR.



- $r \to +\infty$ is the UV, $r \to -\infty$ is the IR.
- Here we chose $L = 1$, $q = 2$, $m^2 = -2$, $u = 3$.
- This solution is essentially unique: related solutions have ψ with nodes.

Null trajectories at constant r have $v(r) \equiv |d\vec{x}/dt| = \sqrt{h(r)}$.

"Index of refraction" $n = v_{\rm UV}/v_{\rm IR} \approx 1.63$ for this setup.

You can also recover Lorentz symmetry but *not* conformal symmetry in IR if $V(|\psi|)$ has no extrema away from $\psi = 0$ [Gubser and Nellore 2009].

3. Embedding in string theory

Focus on AdS_5 embeddings. For AdS_4, see [Denef and Hartnoll 2009].

$\mathcal{N} = 4$ SYM has $SO(6)$ R-symmetry. Let's pick out a $U(1) \subset SO(6)$ by studying states with

$$\langle J_{12} \rangle = \langle J_{34} \rangle = \langle J_{56} \rangle = \frac{\rho}{\sqrt{3}} \,. \tag{10}$$

The AdS_5 dual is the near-horizon limit of spinning D3-branes. The $d = 5$ description is the Reissner-Nordstrom black hole:

$$\begin{gathered}
\mathcal{L} = \frac{1}{2\kappa^2} \left[R - \frac{1}{4} F_{\mu\nu}^2 + \frac{12}{L^2} + (FFA \text{ Chern-Simons}) \right] \\
ds_5^2 = e^{2A}(-h dt^2 + d\vec{x}^2) + \frac{dr^2}{h} \qquad A_{(1)} = \Phi dt \\
A = \frac{r}{L} \qquad h = 1 - \frac{2\epsilon L \kappa^2}{3} e^{-4r/L} + \frac{\rho^2 \kappa^4}{3} e^{-6r/L} \\
\Phi = \rho \kappa^2 (e^{-2r_H/L} - e^{-2r/L})
\end{gathered} \tag{11}$$

Easily calculate $T = \frac{1}{4\pi} e^{A(r_H)} h'(r_H) \qquad \mu = \lim_{r \to \infty} \Phi(r)$.

A convenient definition: $\boxed{\hat{\rho} = \frac{\kappa^2 \rho}{(2\pi L)^3}.}$ $\hat{\rho}^{1/3}$ is a typical energy scale.

A horizon exists iff $\frac{\rho^2}{s^2} \geq \frac{3}{2\pi^2}$, but it has a Gregory-Laflamme instability against making $\langle J_{12} \rangle \neq \langle J_{34} \rangle \neq \langle J_{56} \rangle$ unless [Cvetic and Gubser 1999]

$$\frac{\rho^2}{s^2} \geq \frac{3}{4\pi^2} \qquad \Longleftrightarrow \qquad \frac{T}{\hat{\rho}^{1/3}} \geq \frac{T_{\rm GL}}{\hat{\rho}^{1/3}} = \frac{1}{(48\pi^4)^{1/6}} \approx 0.245\,. \tag{12}$$

It was never settled whether GL is the dominant instability of spinning D3-branes, or what the endpoint of this GL instability is.

The following discussion of superconducting instabilities is based on ongoing work with C. Herzog, S. Pufu, F. Rocha, and T. Tesileanu.

To find $T_c/\hat{\rho}^{1/3}$ for condensation of a charged scalar, we need its charge under $U(1)$ and its mass:

$$m_{\text{eff}}^2 = m^2 + q^2 \Phi^2 g^{tt} \,.$$

At $T = T_c$, there is a static perturbation of $RN\,AdS_5$ by the symmetry-breaking scalar.

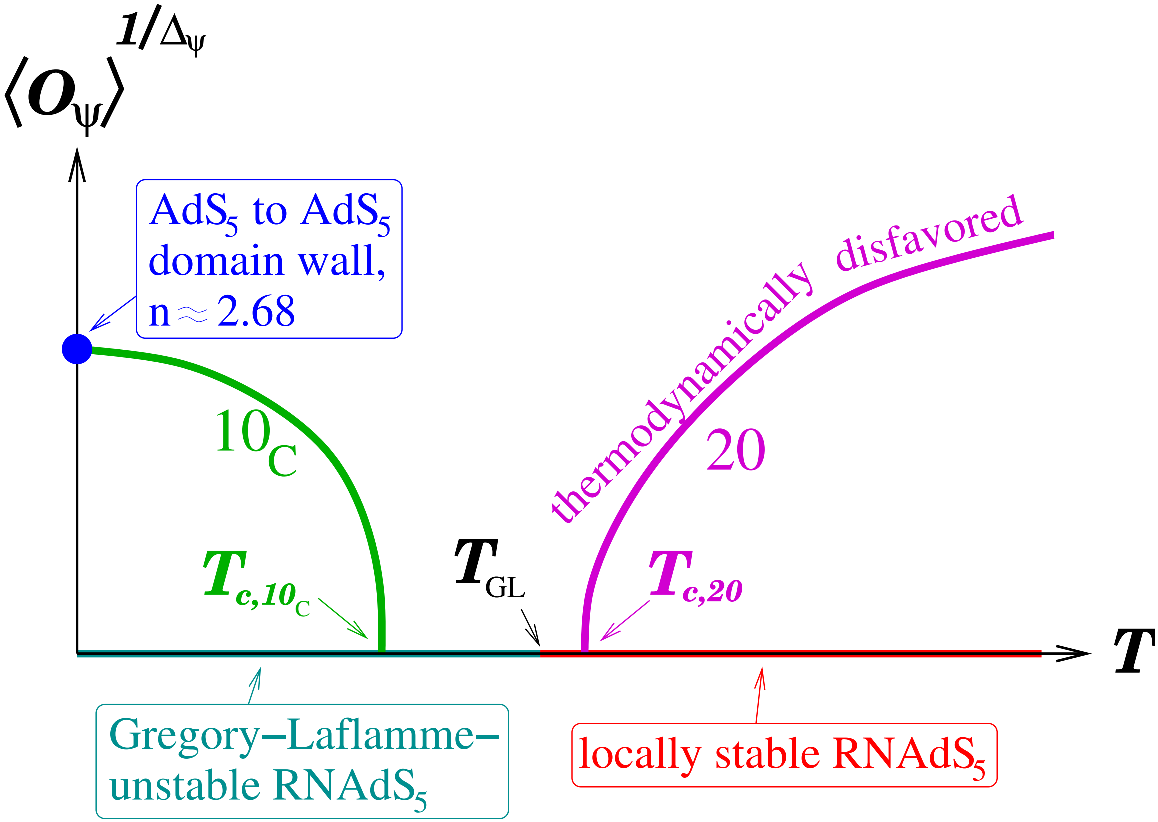
- **20** of $SO(6)$:

  $m^2 L^2 = -4$,
  dual VEV $\langle \text{tr}\, Z_1 Z_2 \rangle$ (for example),
  R-charge $R = 4/3, \quad \Delta = 2$,
  electric charge $qL = 2/\sqrt{3}$.
  $$\Longrightarrow \quad \frac{T_c}{\hat{\rho}^{1/3}} = 0.307$$

- $\mathbf{10_C}$ of $SO(6)$:

  $m^2 = -3/L^2$,
  dual VEV $\langle \text{tr}\, \lambda_4 \lambda_4 + \ldots \rangle$,
  R-charge $R = 2, \quad \Delta = 3$,
  $qL = \sqrt{3}$.
  $$\Longrightarrow \quad \frac{T_c}{\hat{\rho}^{1/3}} = 0.154$$

Different types of symmetry breaking compete:



Must understand supergravity action beyond quadratic order to go to finite $\langle \mathcal{O}_\psi \rangle$.

5-dimensional perspective:

- $\mathbf{20}$, $\mathbf{10_C}$, and $\mathbf{1_C}$ parametrize $E_{6(6)}/USp(8)$ of $d=5$, $\mathcal{N}=8$ SUGRA [Gunaydin et al. 1986]. Uplift to 10-d only partially known.
- Explicit non-linear action and uplift for just the $\mathbf{20}$ plus $SO(6)$ gauge fields is known [Cvetic et al. 2000].
- The $U(1)$ we've selected, plus the highest-charge member of $\mathbf{10_C}$, plus metric are (almost) all the fields in the $SU(3)$-invariant bosonic sector of $d=5$, $\mathcal{N}=8$:

$$\mathcal{L} = R - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}\left[(\partial_\mu \eta)^2 + \sinh^2\eta\left(\partial_\mu\theta - \frac{\sqrt{3}}{L}A_\mu\right)^2\right] + \frac{3}{L^2}\cosh^2\frac{\eta}{2}\left(5-\cosh\eta\right), \tag{13}$$

$\frac{SL(2,\mathbf{R})}{U(1)}$ NLσM

- The non-SUSY vacuum at $\eta = \log(2+\sqrt{3})$ is unstable toward breaking $SU(3)$ [Distler and Zamora 2000].
- A more ornate setup probably flows from $\mathcal{N}=4$ to $\mathcal{N}=1$ superconformal vacuum of [Khavaev et al. 2000], and may be stable.

10-dimensional perspective:

- It helps to view S^5 as a $U(1)$ fibration over $\mathbf{CP}^2$. All results I'll show generalize to SE_5's obtained by replacing $\mathbf{CP}^2$ by a different Einstein-Kahler 2-fold.
- Main trick is to establish some explicit uplift of a sub-theory of $d = 5, \mathcal{N} = 8$ SUGRA to type IIB.

To uplift any solution $(ds_M^2, A_{(1)})$ to $\mathcal{L} = R - \frac{1}{4}F_{\mu\nu}^2 + \frac{12}{L^2} + \mathbf{C.S.}$, use [Cvetic et al. 1999 2000]

$$ds_{10}^2 = ds_M^2 + L^2 \sum_{i=1}^3 |Dz_i|^2 \qquad \sum_{i=1}^3 |z_i|^2 = 1 \qquad Dz_i \equiv dz_i + \frac{i}{L} A_{(1)} z_i$$
$$F_{(5)} = \mathcal{F}_{(5)} + *\mathcal{F}_{(5)} \qquad \mathcal{F}_{(5)} = -\frac{4}{L}\,\text{vol}_M + L^2 (*_M F_{(2)}) \wedge \omega_{(2)} \,. \tag{14}$$

Now generalize to capture superconducting solutions. $SU(3)$ symmetry means we can't squash the $\mathbf{CP}^2$; we can only stretch the $U(1)$ fiber:

$$ds_5^2 = L^2 \left(ds_{\mathbf{CP}^2}^2 + \cosh^2 \frac{\eta}{2} \zeta_{(1)}^2 \right) \qquad \zeta_{(1)} = \frac{i}{2} \sum_{i=1}^3 (z_i d\bar{z}_i - \bar{z}_i dz^i) \tag{15}$$

Including spin: $dz_i \to Dz_i \quad \Longrightarrow \quad \zeta_{(1)} \to \zeta_{(1)}^A \equiv \zeta_{(1)} + \frac{1}{L} A_{(1)}$.

The complex scalar $(\eta, \theta) \in \mathbf{10}_{\mathbf{C}}$ describes deformations sourced by $F_{(2)} \equiv B_{(2)} + iC_{(2)}$. A tricky point: How do we choose $F_{(2)}$?

- Consider the CY_3 cone over our SE_5: $ds^2_{CY_3} = dr^2 + r^2 ds^2_{SE_5}$.
- Normalize holomorphic three-form $\Omega_{(3)}$ so that $\Omega_{(3)} \wedge \Omega^*_{(3)} = 8\,\mathrm{vol}_{CY_3}$. $\Omega_{(3)} = dz^1 \wedge dz^2 \wedge dz^3$ when $CY_3 = \mathbf{C}^3$.
- Decompose $\Omega_{(3)} = r^2 dr \wedge \Omega_{(2)} + \text{(3-form on base)}$
- $\boxed{F_{(2)} = iL^2 e^{i\theta} \tanh\frac{\eta}{2}\,\Omega_{(2)}}$

(Related heavy lifting: [Corrado et al. 2002; Pilch and Warner 2001 2002]; also [Romans 1985])

After some further thought, find

$$ds^2_{(10)} = \cosh\frac{\eta}{2} ds^2_M + \frac{L^2}{\cosh\frac{\eta}{2}} ds^2_5$$

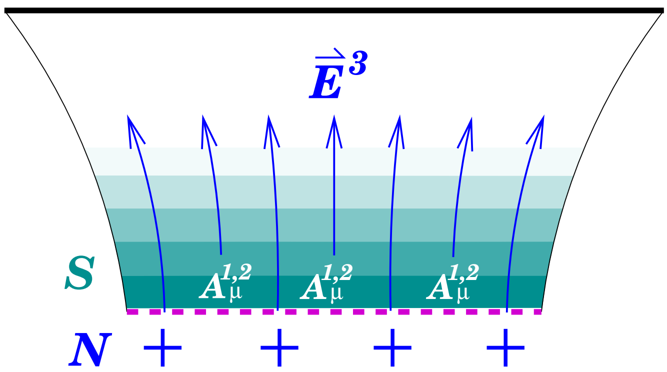
$$\begin{aligned}\mathcal{F}_{(5)} &= \cosh^2\frac{\eta}{2}\,\frac{\cosh\eta - 5}{L}\,\mathrm{vol}_M + L^2(*_M F_{(2)}) \wedge \omega_{(2)} \\ &\quad + L^4 \tanh^2\frac{\eta}{2}\left(d\theta - \frac{3}{L}A_{(1)}\right) \wedge \omega_{(2)} \wedge \omega_{(2)}\end{aligned} \tag{16}$$

4. p-wave superconducting black holes

The ejection of charged particles from a black hole doesn't especially depend on spin. So we'd expect superconducting black hole solutions to

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2} - \frac{1}{4}(F^a_{\mu\nu})^2\right) \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu \,. \quad (17)$$

- Assume that A^3_μ carries the charge: selects $U(1)_3 \subset SU(2)$.
- A^1_μ and/or A^2_μ provide the condensate, spontaneously breaking $U(1)_3$.
- Only meaningful dimensionless parameter in $\mathcal{L}$ is gL.



Two natural guesses for $A^{1,2}_\mu$ [Gubser 2008c; Gubser and Pufu 2008; Roberts and Hartnoll 2008]:

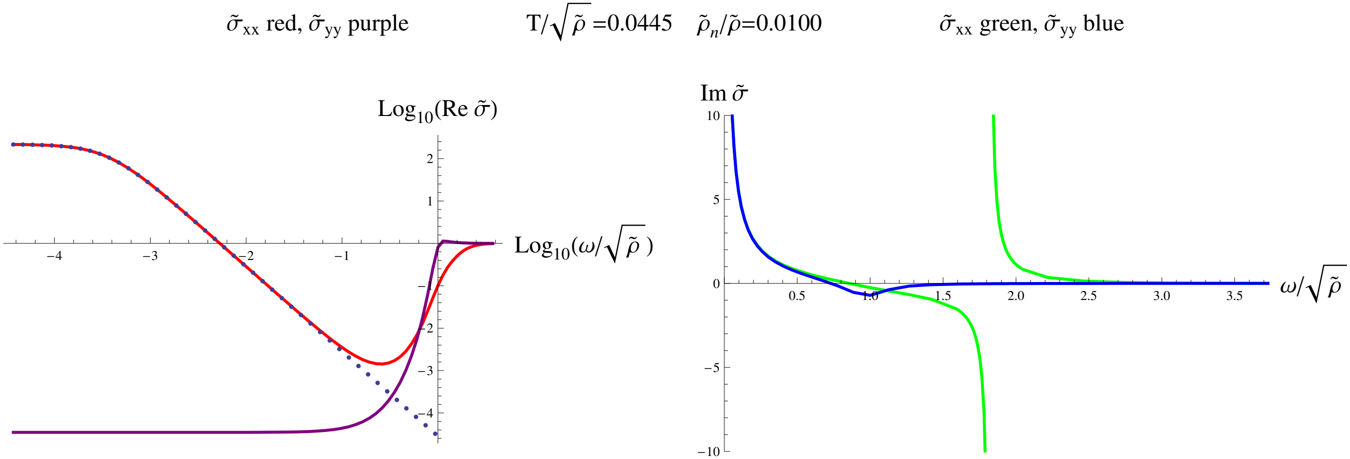
$$\text{“}p + ip\text{-wave:”} \qquad A_{(1)} = \Phi(r)\tau^3 dt + w(r)(\tau^1 dx_1 + \tau^2 dx_2)$$

$$\text{“}p\text{-wave:”} \qquad A_{(1)} = \Phi(r)\tau^3 dt + w(r)\tau^1 dx_1 \,.$$

At least in limit of large gL, non-linear terms in $F^2_{\mu\nu}$ prefer p over $p + ip$.

For AdS_4:

- $\begin{cases} T_c/\mu \to \infty & \text{as} \quad gL \to \infty \\ T_c/\mu \to 0 & \text{at} \quad gL = g_c L \approx 0.8\,. \end{cases}$
- For large gL, define $\tilde{\rho} = \kappa^2 g L^2 \rho$. $\sqrt{\tilde{\rho}}$ is a typical energy scale.
- Conductivity is anisotropic: $\sigma_{mn}(\omega) = \frac{i}{\omega} G^R_{mn}(\omega, 0)$ where $G^R_{mn}(t, \vec{x}) = -i\theta(t)\langle [J_m(t,\vec{x}), J_n(0,0)]\rangle$. For large gL, find this:



For AdS_5: $g_c L = 1$. So equal-charge spinning D3's *just* avoid this instability.

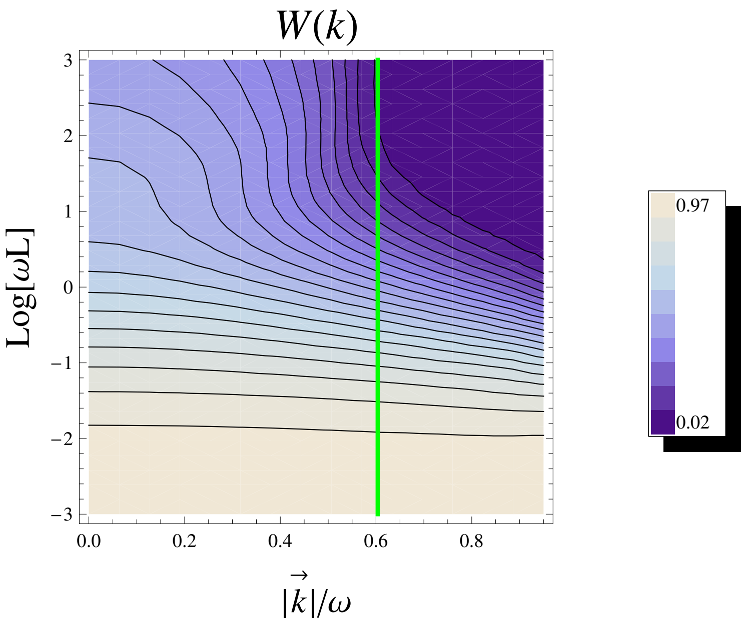
5. Are superluminal unparticles implausible?

Could we arrange a string compactification where observed c is $v_{\rm IR}$, while an "unparticle" sector [Georgi 2007] has $v_{\rm UV} = h_{\rm UV} v_{\rm IR}$ with $h_{\rm UV} = n^2 > 1$? [Gubser 2008a], c.f. [Kiritsis 1999; Alexander 2000; Csaki et al. 2001].

- Unparticle Green's functions have spectral measure that concentrates within a *narrower* momentum-space light-cone above some energy scale $1/L$.
- In real-space, UV light-cone is therefore broader.
- I show a contour plot of

$$W(\omega, \vec{k}) \equiv \frac{\operatorname{Im} G_F(\omega, \vec{k})}{\operatorname{Im} G_F^{\rm IR}(\omega, \vec{k})}$$

  in an example where $\Delta = 3.98$ and $n \approx 1.7$.
- Green line is UV-null.

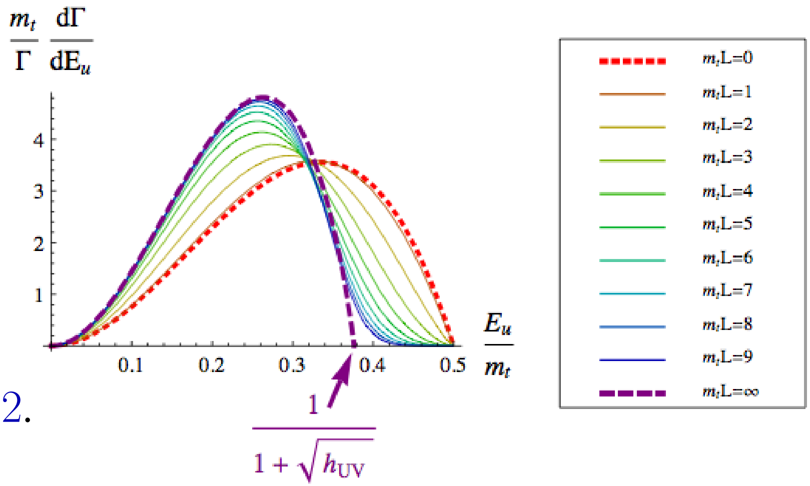


There are difficulties both phenomenological and theoretical [Cline and Valcarcel 2004; Gubser 2008a] with compactifications involving FTL regions: null energy violation, normalizable graviton, stringent experimental bounds on universality of c....

But there is a striking direct-detection signature based on a heavy-to-light decay $t \to u + \mathcal{U}$, where $\mathcal{U}$ is superluminal unparticle stuff:

$$\mathcal{L}_{\rm int} = i \frac{\lambda}{\Lambda^{\Delta_{\mathcal{O}}}} \bar{u} \gamma_\mu (1 - \gamma_5) t \, \partial^\mu \mathcal{O} + \text{h.c.}$$

- $m_t = E_u + E_{\mathcal{U}}$ and $0 = \vec{p}_u + \vec{p}_{\mathcal{U}}$.
- On-shell, $E_u = |\vec{p}_u|$.
- Unparticles have $E_{\mathcal{U}} \geq \sqrt{h_{\rm UV}} |\vec{p}_{\mathcal{U}}|$.
- Combine to get $E_u \leq m_t/(1 + \sqrt{h_{\rm UV}}) < m_t/2$.



The explicit curves are based on previously described $W(\omega, \vec{k})$.

6. Conclusions

The motivating questions:

1. How is the black hole "no hair theorem" violated?
2. What happens to N coincident D3-branes (or M2-branes, or branes on singularities...) when you give them some spin in the compact directions?
3. If we follow our noses with simple constructions in supergravity and string theory, how close do we get to real-world superconductors?

Some provisional answers:

1. Through some combination of gauge fields, non-renormalizable interactions, and/or AdS asymptotics.
2. Several symmetry-breaking instabilities compete, and $O(1)$ factors from supergravity dynamics matter.
3. We do *not* have lattice structure, which seems crucial to many modern theoretical attempts. But phase competition among multiple instabilities actually seems to me in the correct ballpark.

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