Holographic superconductors

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Plan of talk

Motivation

1. What is a superconductor?
2. What does a typical theory of superconductivity look like?
3. What defines a nonconventional superconductor?

Holographic superconductors

1. Ingredients for a holographic superconductor
2. Black hole instabilities
3. Electrical conductivity
4. Landscape of superconducting membranes

Motivation

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What is a superconductor?

- Spontaneously broken global $U(1)$ symmetry
  $\Rightarrow$ Goldstone boson with transformation: $\theta \to \theta + \Lambda$.

- Gauge invariance $\Rightarrow$ free energy in background potential A:

$$F = \int d^d x \, f[A - d\theta] \,.$$

- Current generated by a small field

$$J = - \left. \frac{\delta F}{\delta A} \right|_{A = d\theta + \delta A} = -f''[0] \delta A \qquad \text{(London equation)} \,.$$

- It follows that the electrical conductivity diverges as $\omega \to 0$

$$J = \frac{i f''[0]}{\omega} \delta E \equiv \sigma(\omega) \delta E \,.$$

What does a typical theory of superconductivity look like?

- In a textbook on superconductivity one finds the 'BCS Hamiltonian'

$$H = \sum_{k,\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} - |g_{\text{eff}}|^2 \sum_{k,k'} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-k'\downarrow} c_{k'\uparrow} + \sum_k A_k \cdot J_{-k} \,.$$

- Interaction term is generated by the exchange of a soft phonon between two effective electrons. Need $|\epsilon_k - \epsilon_F|, |\epsilon_{k'} - \epsilon_F| \ll \omega_D$.

- Theory predicts the symmetry breaking condensate

$$\Delta \equiv |g_{\text{eff}}|^2 \langle c_{-k\downarrow} c_{k\uparrow} \rangle = 2\omega_D e^{-1/|g_{\text{eff}}|^2 g(\epsilon_F)} \,.$$

ω_D is Debye frequency (energy scale of phonons) and $g(\epsilon_F)$ density of states at Fermi energy.

- Theory predicts the critical temperature in terms of the condensate

$$\frac{2\Delta}{T_C} = 3.52 \,.$$

- The electrical conductivity in the superconducting phase is computed from the current two point function

$$\sigma(\omega) = \frac{-i\langle J_x J_x\rangle^R(\omega)}{\omega} \,.$$

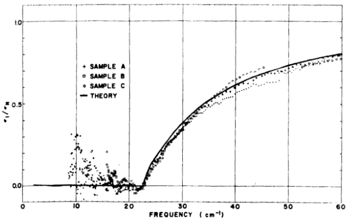
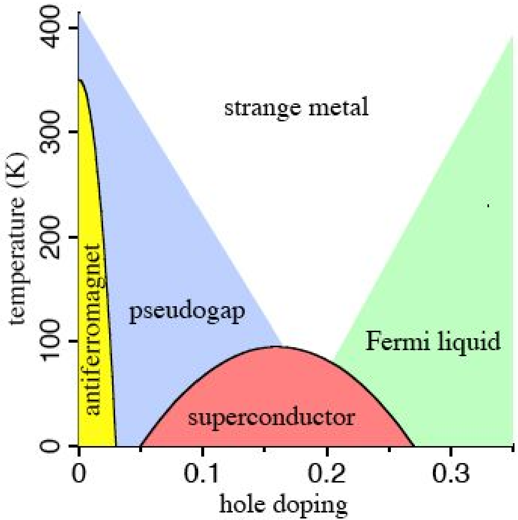


Fig. 2. Measured and calculated values of the conductivity ratio σ_1/σ_n of three superconducting lead films as a function of the photon frequency. [After Palmer (39).]

- The $U(1)$ symmetry is global in BCS theory – photons not important.

Phase diagram of cuprate High - T_c superconductors



Two senses of non-BCS

- Ingredients of BCS theory
  - Weakly coupled excitations: 'glue' (phonons) + dressed electrons.
  - Interactions: 'pairing mechanism'.
- Simple sense of non-BCS
  - The glue is not phonons.
  - Effective theory of electrons similar to BCS.
  - Example: heavy fermion compounds – paramagnons.
- A stronger sense of non-BCS
  - There are no quasiparticles!
  - Strongly interacting 'soup' with spontaneous symmetry breaking.
  - Possibly relevant for High-T_c.
  - Use AdS/CFT as a solvable example.

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Minimal ingredients for a holographic superconductor

- Minimal ingredients
  - Continuum theory ⇒ have $T^{\mu\nu}$ ⇒ need bulk g_{ab}.
  - Conserved charge ⇒ have J^{μ} ⇒ need bulk A_a.
  - 'Cooper pair' operator ⇒ have $\mathcal{O}$ ⇒ need bulk ϕ.

- Write a minimal 'phenomenological' bulk Lagrangian

$$\mathcal{L}_{1+3} = \frac{1}{2\kappa^2} R + \frac{3}{L^2 \kappa^2} - \frac{1}{4g^2} F_{ab} F^{ab} - |\nabla \phi - iqA\phi|^2 - m^2 |\phi|^2 \,.$$

There are four dimensionless quantities in this action.

- Newton's constant ⇒ central charge of the CFT: $c = 192 L^2/\kappa^2$.
- Maxwell coupling ⇒ DC conductivity $\sigma_{xx} = \frac{1}{g^2}$.
- Mass ⇒ scaling dimension $\Delta(\Delta - 3) = (mL)^2$.
- Charge q is the charge of the dual operator $\mathcal{O}$.

Two instabilities of a charged AdS black hole

- By dimensional analysis $T_c \propto \mu$.
- The dual geometry is therefore Reissner-Nordstrom-AdS.
- RN-AdS can be unstable against a (charged) scalar for two reasons.
- Reason 1 [Gubser '08]: Background charge shifts mass:

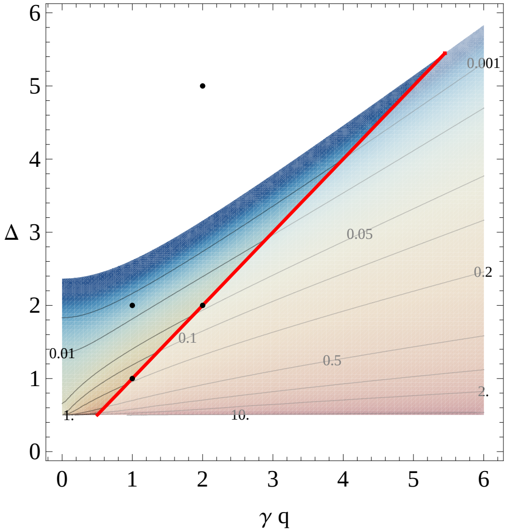
$$m^2_{\text{eff.}} \sim m^2 - q^2 A_t^2 \,.$$

- Reason 2 [SAH-Herzog-Horowitz '08]: Near extremality AdS_2 throat with

$$m^2_{\text{BF-2}} = -\frac{1}{4L_2^2} = -\frac{3}{2L^2} > -\frac{9}{4L^2} = m^2_{BF-4} \,.$$

- Precise criterion for instability at $T = 0$ [Denef-SAH '09, Gubser '08]

$$q^2\gamma^2 \geq 3 + 2\Delta(\Delta - 3)\,, \qquad \gamma^2 = \frac{2g^2L^2}{\kappa^2}\,.$$



[Denef-SAH '09]

Endpoint – hairy black holes

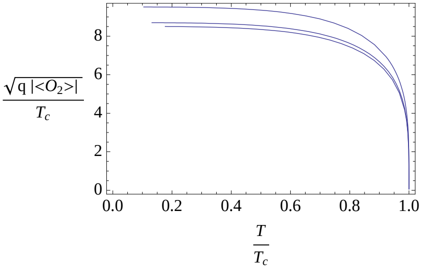
[SAH-Herzog-Horowitz '08]

- Endpoint of instability is a hairy black hole:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + \frac{L^2}{r^2}\left(dx^2 + dy^2\right) \, ,$$

$$A = A_t(r)dt \, , \qquad \phi = \phi(r) \, .$$

- Solve numerically (take $\Delta = 2$). Can obtain $\langle \mathcal{O} \rangle$:

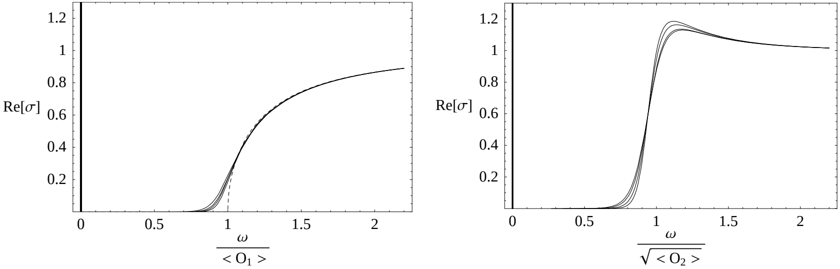


- Compare 8 to ~ 3.5 for BCS and $\sim 5 - 8$ for High-T_C.

Electrical conductivity

[SAH-Herzog-Horowitz '08]

- Computed the conductivity. At $T \sim 0$, typical curves



- If the gap is 2Δ then we found that

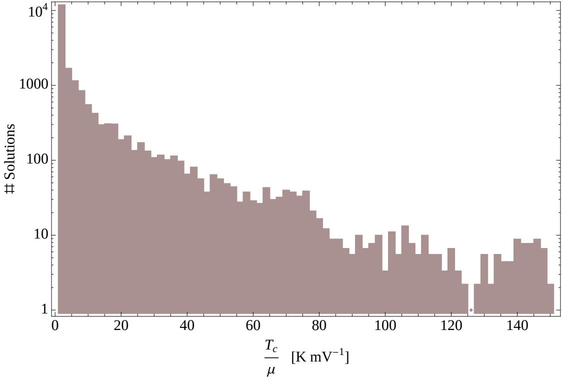
$$\mathrm{Re}\,\sigma(\omega \to 0) \sim e^{-\alpha\Delta/T}.$$

- Generally $\alpha \neq 1$, unlike BCS theory, no weakly coupled picture in terms of Cooper 'pairs'.
- Exact gap means the Goldstone boson is not contributing.

Landscape of superconducting membranes

[Denef-SAH '09]

- Many examples in Sasaki-Einstein compactifications of M theory.
- Distribution of critical temperatures



Probing the normal state with magnetic fields

[Denef-SAH-Sachdev *in preparation*]

- The secret of superconductivity lies in the normal state.
- The charged-AdS black hole is dual to a new exotic state of matter.
- It is not a weakly coupled 'Fermi liquid'.
- What is it?
- Magnetic fields are a powerful tool to characterise Fermi surfaces: de Haas-van Alphen oscillations.
- Looking for these in AdS/CFT requires one loop in the bulk ($1/N$).

Summary

- Theories of superconductivity: weak coupling 'pairing' of electrons.
- May not apply to nonconventional superconductors, e.g. cuprates.
- AdS/CFT: strongly coupled system exhibiting superconductivity.
- Showed results for condensate and electrical conductivity.
- $\exists$ 'landscape' of superconductors in M theory.
- Key question: how to think about the 'normal' state?