On the gravity duals of $\mathcal{N}$=4 Chern-Simons theories

(monopole operators and wrapped M2-branes)

2009/6/23(Tue)
Strings2009@Rome

Yosuke Imamura
Tokyo Univ.

Based on: arXiv:0812.1331 Y.I. and S.Yokoyama
arXiv:0902.4173 Y.I.
arXiv:0907.xxxx Y.I. and S. Yokoyama

Introduction

In this talk, I'd like to discuss AdS_4/CFT_3.

On this topic, there was big progress in the last two years, which had been triggered by the discovery of interacting Chern-Simons matter system with $\mathcal{N}=8$ supersymmetry (BLG model).

After the discovery, various CSs with different $\mathcal{N}$ have been constructed.

$\mathcal{N}=8$ (BLG model)	Bagger-Lambert, Gusstavson
$\mathcal{N}=6$ (ABJM model)	Aharony-Bergman-Jafferis-Maldacena
$\mathcal{N}=4$	Gaiotto-Witten, Hosomichi-Lee-Lee-Lee-Park

In $\mathcal{N}\leqq 6$ theories, we can take the large N_c limit, and consider AdS/CFT.

AdS_4/CFT_3 claims the equivalence between

M-theory in $AdS_4 x X_7$ ⇔ 3-dim CFT

For various internal space X_7, dual CFTs (Chern-Simons theories) are proposed.

$X_7 = S^7/Z_k$ $\mathcal{N}=6$ CS theory (ABJM model)

$X_7 = S^7/\Gamma$ $\mathcal{N}=4$ CS theories
(Γ : certain discrete groups)

$X_7 = SE_7$ $\mathcal{N}=2$ quiver CS theories

How can we confirm these dualities?

The field operator correspondence gives non-trivial evidence for the equivalence.

In general, it claims the correspondence

Fields in AdS_{d+1} $\longleftrightarrow$ Local operators in CFT_d

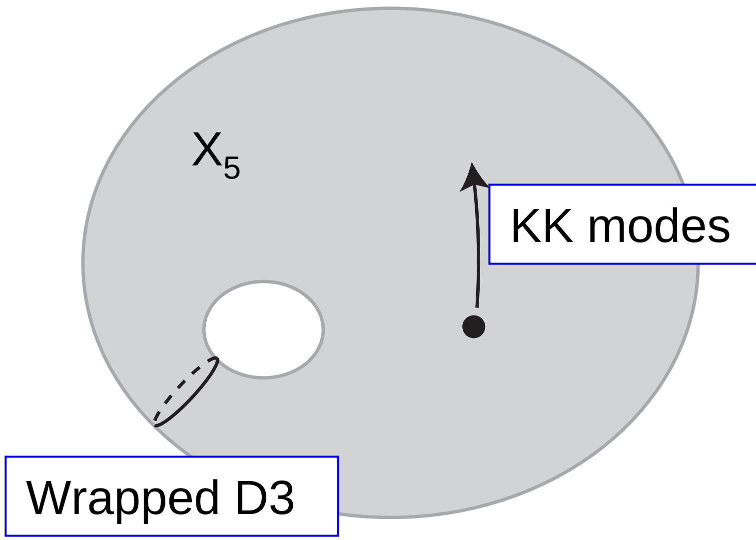
Once we have established this correspondence, we can compute gauge theory correlation functions on the gravity side (in a certain parameter region).

Example: 2-pt functions

$$\Delta = R_{\mathrm{AdS}} m \qquad \langle O(x) O(y) \rangle = \frac{1}{|x-y|^{2\Delta}}$$

In the case of AdS_5/CFT_4, there are two kinds of fields in AdS_5

Type IIB / $AdS_5 x X_5$ ⟷ Boundary CFT_4



Small (mesonic) operators

Tr(AB) etc.

$$\Delta \sim \mathcal{O}(1)$$

[Witten, hep-th/9802150]

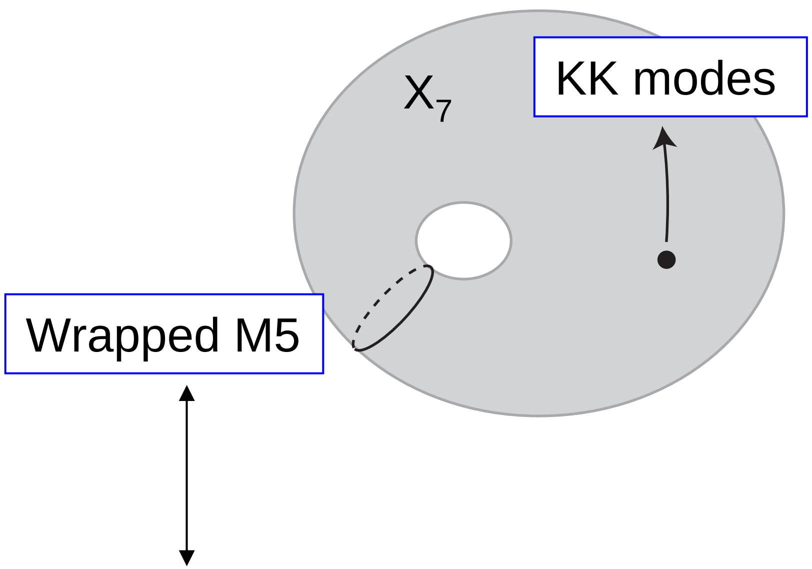
Large (baryonic) operators

Det(X) etc. $\Delta \sim \mathcal{O}(N_c)$

[Gubser-Klebanov, hep-th/9808075]
[Gukov-Rangamani-Witten, hep-th/9811048]

We have similar correspondence in the case of AdS_4/CFT_3

M-theory / $AdS_4 x X_7$



Small (mesonic) operators

Tr(AB) etc.

$$\Delta = R_{\mathrm{AdS}} m \sim \mathcal{O}(1)$$

Large (baryonic) operators

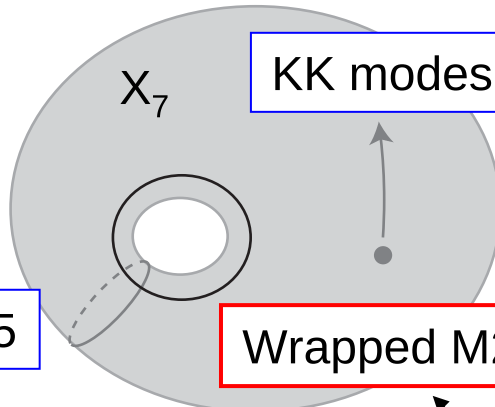
Det(X) etc.

$$\Delta = R_{\mathrm{AdS}} m \sim \mathcal{O}(N_c)$$

[C.S.Park, arXiv:0810.1075](ABJM)
[Y.I. and S.Yokoyama, arXiv:0812.1331]($\mathcal{N}$=4)

In AdS_4/CFT_3 we have another type of objects.

M-theory / $AdS_4 x X_7$



KK modes ↔ Small (mesonic) operators

Tr(AB) etc.

$$\Delta = R_{\rm AdS} m \sim \mathcal{O}(1)$$

Wrapped M5 ↔ Large (baryonic) operators

Det(X) etc.

$$\Delta = R_{\rm AdS} m \sim \mathcal{O}(N_c)$$

[C.S.Park, arXiv:0810.1075](ABJM)
[Y.I. and S.Yokoyama, arXiv:0812.1331]($\mathcal{N}$=4)

Wrapped M2 ↔ ? ? ?

What are operators corresponding to wrapped M2-branes?

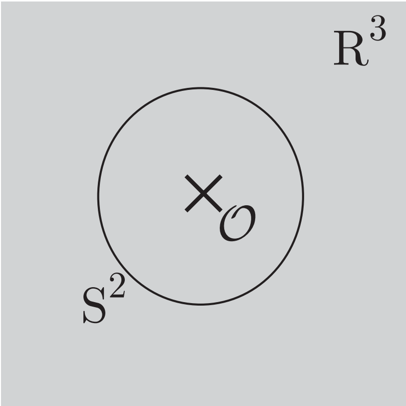
Our proposal:

They correspond to (a part of) monopole operators.

[arXiv:0812.1331 Y. I. and S.Yokoyama]

[arXiv:0902.4173 Y. I.]

Monopole operators = Operators carrying magnetic charge



$$m = \frac{1}{2\pi} \oint_{\mathbf{S}^2} F$$

Note:

No such a question arises for ABJM model, because

$$b_2(\mathbf{S}^7/\mathbf{Z}_k) = 0$$

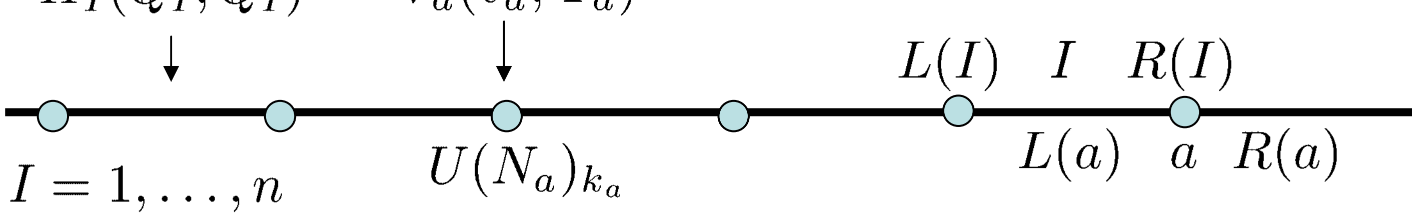
We should consider $\mathcal{N} \leqq 4$ theories.

It seems nice starting point to investigate the field-operator correspondence for wrapped M2-branes in $\mathcal{N}=4$ case in which some quantum corrections on the field theory side are suppressed.

$\mathcal{N}$=4 Chern-Simons theories

An $\mathcal{N}$=4 CS theory is described by a circular quiver diagram

[Hosomichi et al.,arXiv:0805.3662]



Edges : $I = 1, \ldots, n$

Vertices : $a = 1, \ldots, n$

Action (similar to D=4 $\mathcal{N}$=2 SQCD)

$$S_{\rm kin} = \sum_{I=1}^{n} \int d^3x d^4\theta {\rm tr}(e^{-v_{R(I)}} Q_I^\dagger e^{v_{L(I)}} Q_I + e^{-v_{R(I)}} \widetilde{Q}_I e^{v_{L(I)}} \widetilde{Q}_I^\dagger)$$

$$S_{\rm pot} = \sum_{I=1}^{n} \int d^3x d^2\theta {\rm tr}(\widetilde{Q}_I \Phi_{L(I)} Q_I - \widetilde{Q}_I Q_I \Phi_{R(I)}) + {\rm c.c.}$$

$$S_{\rm CS} = \sum_{a=1}^{n} \frac{k_a}{2\pi} \int d^3x d^4\theta {\rm tr}(v_a D e^{tv_a} \overline{D} e^{-tv_a}) + \sum_{a=1}^{n} \frac{k_a}{2} \left(\int d^3x d^2\theta \Phi_a^2 + {\rm c.c.} \right)$$

Chern-Simons terms

$$S_{\rm CS} = \sum_{a=1}^{n} \frac{k_a}{4\pi} \int \mathrm{tr}\left(A_a dA_a + \frac{2}{3} A_a^3\right) + \cdots$$

$k_a \in Z$: Chern-Simons levels

$\mathcal{N}=4$ SUSY requires the Chern-Simons levels to be

$k_a = k(s_{L(a)} - s_{R(a)})$

$s_I=0$ or 1. $\{s_1,s_2\}=\{0,1\}$ → ABJM model

Corresponding to two values of s_I, there are two kinds of hypermultiplets.

$s_I=0$: untwisted hyper [Hosomichi et al., arXiv:0805.3662]

$s_I=1$: twisted hyper

We use $i=1,\ldots,p$ and $i'=1,\ldots,q$ for untwisted and twisted hypermultiplets.

p: # of untwisted hypermultiplet

q: # of twisted hypermultiplet

$p+q=n$

Global symmetries

R-symmetry : Spin(4) = $SU(2)_A \times SU(2)_B$

Flavor symmetries: U(1) × U(1)'

	$SU(2)_A$	$SU(2)_B$	U(1)	U(1)'
Untwisted hyper $(q_i, \widetilde{q}_i^\dagger)$	2	1	1	0
Twisted hyper $(q_{i'}, \widetilde{q}_{i'}^\dagger)$	1	2	0	1

Moduli space

The background geometry for a single M2-brane is determined as the Higgs branch moduli space of N_c=1 theory

Moduli space = Solution of F-term conditions / Complexified gauge sym $U(1)_c^{n-1}$

By using F-term conditions and complexified gauge symmetry, we can set

$$q_{i=1} = \cdots = q_{i=p}$$

$$\widetilde{q}_{i=1} = \cdots = \widetilde{q}_{i=p}$$

$$q_{i'=1} = \cdots = q_{i'=q}$$

$$\widetilde{q}_{i'=1} = \cdots = \widetilde{q}_{i'=q}$$

$e^{ia+\sigma} = 1$ (complexified dual photon field)

ϕ_a — dependent fields

The continuous part of gauge symmetry is fixed.

There are four independent variables (coordinates in the moduli space)

$$(z_1, z_2, z_3, z_4) = (q_i, \widetilde{q}_i^\dagger, q_{i'}, \widetilde{q}_{i'}^\dagger)$$

Action of global symmetries

	z_1	z_2	z_3	z_4
$e^{i\epsilon T_3} \in SU(2)_A$	$e^{i\epsilon/2}$	$e^{-i\epsilon/2}$	1	1
$e^{i\epsilon T_3'} \in SU(2)_B$	1	1	$e^{i\epsilon/2}$	$e^{-i\epsilon/2}$
$e^{i\epsilon P} \in U(1)$	$e^{i\epsilon}$	$e^{i\epsilon}$	1	1
$e^{i\epsilon P'} \in U(1)'$	1	1	$e^{i\epsilon}$	$e^{i\epsilon}$

Residual gauge symmetries

$$(z_1, z_2, z_3, z_4) \to \exp\left(\frac{2\pi i}{p} P\right)(z_1, z_2, z_3, z_4)$$

$$(z_1, z_2, z_3, z_4) \to \exp\left(\frac{2\pi i}{q} P'\right)(z_1, z_2, z_3, z_4)$$

$$(z_1, z_2, z_3, z_4) \to \exp\left(\frac{2\pi i}{kq} P' - \frac{2\pi i}{kp} P\right)(z_1, z_2, z_3, z_4)$$

These defines the orbifold

Moduli space = $((\mathbf{C}^2/\mathbf{Z}_p) \times (\mathbf{C}^2/\mathbf{Z}_q))/\mathbf{Z}_k = \mathbf{C}^4/\Gamma$

The internal space X_7 is

$$X_7 = (\mathbf{C}^4/\Gamma)|_{r=1} = \mathbf{S}^7/\Gamma$$

Orbifold of 7-sphere

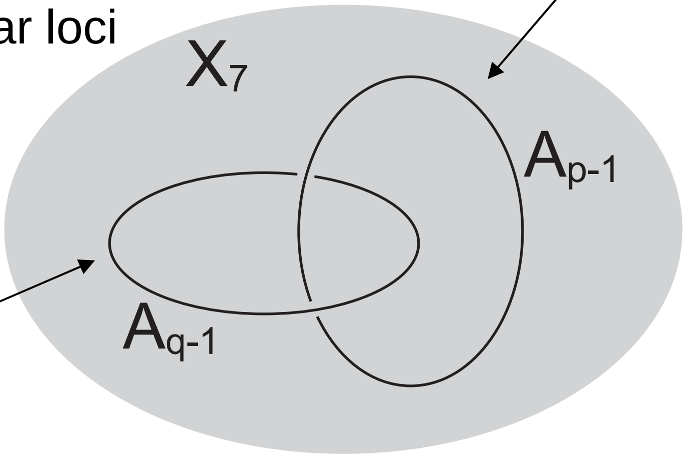
Internal space X_7

$$X_7 = (\mathbf{S}^7/(\mathbf{Z}_p \times \mathbf{Z}_q))/\mathbf{Z}_k$$

There are two singular loci

Z_p fixed submfd ($z_1=z_2=0$)

(p-1) 2-cycles



Z_q fixed submfd ($z_3=z_4=0$)

(q-1) 2-cycles

There are p+q-2 independent 2-cycles

M2-branes can wrap on these cycles.

What are the dual operators in the CS theory?

→ (a part of) Monopole operators

Preliminary check

As a necessary condition, let us confirm the agreement between the numbers of charges

On the field theory side, we have p+q U(N) gauge groups

There are p+q magnetic charges corresponding to p+q diagonal U(1) gauge groups

$$m_a = \frac{1}{2\pi} \oint \mathrm{tr}\ F_a$$

The # of magnetic charges is greater than $b_2=p+q-2$ by 2

Disagreement?

Equation of motion of A_D (The diagonal U(1) decoupled from matter fields)

$$\sum_{a=1}^{p+q} k_a \mathrm{tr}\ F_a = 0. \xrightarrow{\int_{\mathbf{S}^2}} \sum_{a=1}^{p+q} k_a m_a = 0.$$

# of independent magnetic charges = p+q-1

e^{ia} is a monopole operator carrying the $U(1)_D$ magnetic charge.

Because the dual photon field a is regarded as the coordinate of the ``11-th direction'', the charge of this operator should be identified with the corresponding KK-momentum (D-particle charge).

$U(1)_D$ magnetic charge = bulk KK momentum

of ``non-diagonal'' magnetic charges = p+q-2

This agrees with the number of 2-cycles!!

p+q-1 Mag. charges	One diagonal	⟷	KK momentum along ``M''-direction (D-particle #)
	p+q-2 Non-diagonal	⟷	M2-brane wrapping #s.

More Detailed analysis

A Witten Index

$$I(x,z,z') = \mathrm{Tr}\left[(-)^F e^{-\beta'\{Q,S\}} x^{\Delta+R} z^P z'^{P'}\right]$$

Δ : Conformal dimension

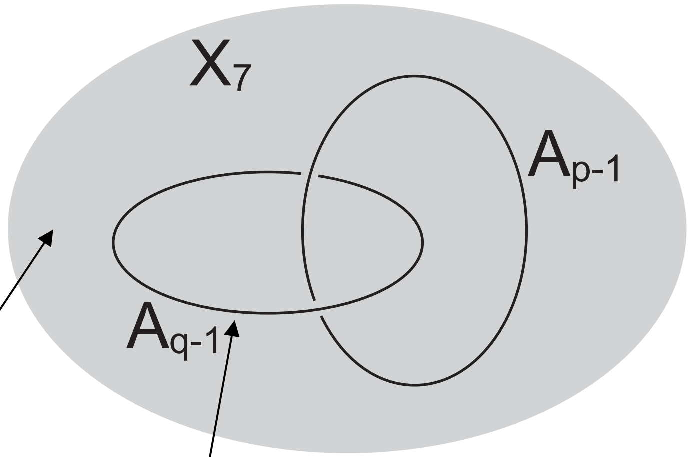
$R=T_3+T_3'$: ($\mathcal{N}=2$) R-charge

P, P' : U(1), U(1)' charges

This should agree with a similar multi-particle index for graviton KK modes.

(non-diagonal) Monopole contribution $\overset{?}{=}$ wrapped M2 contribution

The multi-particle index in $AdS_4 x X_7$ consists of several parts.



SU(q) vector multiplet localized at A_{q-1}

SU(p) vector multiplet localized at A_{p-1}

Bulk KK modes	Cartan part	Cartan part
	+	+
	Non-Cartan part	Non-Cartan part

These parts are identified with wrapped M2 branes

These parts are reproduced as gauge theory index without taking account of monopole operators.[Choi-Lee-Song,arXiv0811.2855]

A gauge theory index including monopole contribution for ABJM model is computed in [C. Kim, arXiv.0903.4172], and the agreement with the multi-graviton index in $AdS_4 x(S^7/Z_k)$ [Bhattacharya et al., arXiv:0801.1435] is confirmed.

In a work in progress (with S.Yokoyama), we are computing the gauge theory index for $\mathcal{N}$=4 theories with taking account of the monopole contribution.

The gauge theory index seems to coinside with the multi-particle index including the wrapped M2 contribution (at least up to the order we have checked).

This is a strong evidence for the correspondence

(non-diagonal) Monopole operators ⇔ wrapped M2-branes

Conclusions

We proposed a relation:

Wrapped M2-branes ⇔ (a part of) monopole operators

We confirmed that

$b_2(X_7)$= # of ``non-diagonal'' monopole charges

A gauge theory index agree with a multi-particle index including the contributions of wrapped M2.

[Y. I. and S. Yokoyama, arXiv:0907.xxxx]

Remaining problems

- Extention to other theories ($\mathcal{N} \leqq 3$)

- Sasaki-Einstein manifolds ($\mathcal{N}=2$) with non-vanishing 2-cycles
  → $\Delta \sim N^{1/2}$ (middle size operators?)

- Roles in dualities among 3d CFTs.

- …