M2 branes, CS theories and AdS4/CFT3

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Outline

- Introduction
- Monopoles and quantum corrections
- Host of examples, N=8,6,5,4,3,2,1,0
- Integrability
- Condensed matter applications

Motivations

- Understanding the mysterious M2 brane
- CFT duals for the Landscape
- Physics of 3d quantum field theory
- Possibly a new exactly solvable theory
- Condensed matter applications

History

- It was believed that since the M2 brane theory is strongly coupled, there existed no Lagrangian description of the IR fixed point.
- Schwarz suggested that a Chern-Simons-matter theory could do the job, since it has exactly marginal couplings, unlike the dimensionful Yang-Mills coupling constant in 3d. However, no CSM theory with more than N =3 supersymmetry was thought possible.
- There were some attempts to find less supersymmetric AdS4/CFT3 duals, but without the guide of the M2 theory in flat space, none were completely successful.

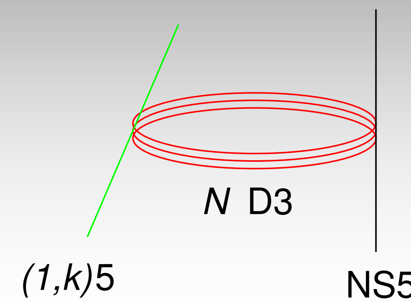
Bagger-Lambert-Gustavsson

- Bagger, Lambert and Gustavsson found a Lagrangian 2+1 conformal field theory with N=8 supersymmetry, based on a 3-Lie algebra, which turned out to be essentially unique. It was reformulated as an $SU(2)_k \times SU(2)_{-k}$ CSM matter theory.
  [van Raamsdonk]

- Its precise physical interpretation remains unclear – perhaps the relation of the moduli space of the level 3 theory with that of G_2 Yang-Mills is a clue. For k=2 it is believed to describe 2 M2 branes on a Z_2 orbifold.

[Distler Mukhi Papageorgakis van Raamsdonk, Lambert Tong]

- A class of N=4 CSM theories were then discovered in Gaiotto Witten and generalized by Hosomichi Lee Lee Lee Park.



- T-dualizing and lifting to M-theory results in N M2 branes probing C^4/Z_k described by a $U(N)_k \times U(N)_{-k}$ $\mathcal{N}$=6 CSM theory. The rg flow from $\mathcal{N}$=3 YM-CS theory just erases the YM term.

[Aharony Bergman Maldacena DLJ]

- Now a large zoo of examples, that I will try to tame somewhat in this talk.

Why is there an M2 brane Lagrangian?

A background in which they become weakly coupled was found, due to the presence of a small circle.

Moreover, reducing to IIA along the natural U(1) isometry results in a background in which the black D2 brane solution has a smooth AdS near horizon limit.

D2 branes

- The effective worldvolume gauge theory on N D2 branes is the N=8 super Yang-Mills. This is the dimensional reduction of the N=4 theory.
- This theory is not conformal; the Yang-Mills coupling is dimensionful in 2+1 dimensions.
- Competing non-renormalization theorems: Cubic superpotential should not renormalize, but neither should hypermultiplets get anomalous dimensions.

Black D2/M2 branes

- The black D2 supergravity solution has SO(7) rotational invariance. It does not have a smooth AdS near horizon limit.

- The string coupling blows up near the D2, so one lifts to M-theory. The black M2 solution in 11d has a smooth $AdS_4 \times S^7$ near horizon geometry.

- The spectrum of chiral operators starts with dimension ½. But these cannot be the same matter fields as in the N=8 YM.

A different IIA reduction

- The $AdS_4 \times S^7/Z_k$ has a natural U(1) isometry, associated to the description of S^7 as a S^1 bundle over CP^3. In the 't Hooft limit one gets IIA on $AdS_4 \times CP^3$ with k units of F_2 flux.

[Nilsson Pope, Volkov Sorokin Tkach]

- This extends to the entire black M2 solution. This gives a background of IIA, with varying dilaton and F_2 flux, in which the black D2 **does** have a smooth near horizon.

- The string coupling is small if k is large.

$$(Nk)^{3/2}/k = \frac{N^2}{(N/k)^{1/2}}$$

Two natural generalizations

- Find other 7d conical backgrounds in IIA with vanishing dilaton at the origin, in which the black D2 brane will have a smooth near horizon.
- Most natural method for M2 branes on susy 8-manifolds.
- Marginally deform the CFT, or follow a relevant operator to a new fixed point, and identify the dual geometry.
- Typically gives vacua with fluxes on the internal manifold.

Chern-Simons-matter theory

- We first consider the case with N=2 susy. It consists of a vector multiplet in the adjoint of the gauge group, and chiral multiplets in representations R_i

$$S_{CS}^{\mathcal{N}=2} = \frac{k}{4\pi} \int (A \wedge dA + \frac{2}{3} A^3 - \bar{\chi}\chi + 2D\sigma)$$

- The kinetic term for the chiral multiplets includes couplings $-\bar{\phi}_i \sigma^2 \phi_i - \bar{\psi}_i \sigma \psi_i$

- There is the usual D term $\bar{\phi}_i D \phi_i$

We integrate out D, σ, and χ

$$S^{\mathcal{N}=2} = \int \frac{k}{4\pi}(A \wedge dA + \frac{2}{3}A^3) + D_\mu \bar{\phi}_i D^\mu \phi_i + i\bar{\psi}_i \gamma^\mu D_\mu \psi_i$$

$$-\frac{16\pi^2}{k^2}(\bar{\phi}_i T^a_{R_i} \phi_i)(\bar{\phi}_j T^b_{R_j} \phi_j)(\bar{\phi}_k T^a_{R_k} T^b_{R_k} \phi_k) - \frac{4\pi}{k}(\bar{\phi}_i T^a_{R_i} \phi_i)(\bar{\psi}_j T^a_{R_j} \psi_j)$$

$$-\frac{8\pi}{k}(\bar{\psi}_i T^a_{R_i} \phi_i)(\bar{\phi}_j T^a_{R_j} \psi_j).$$

Note that this action has classically marginal couplings. It is has been argued that it does not renormalize, up to shift of k, and so is a CFT.

N=3 CS-matter

- To obtain a more supersymmetric theory, begin with N=4 YM-matter. Then add the CS term, breaking to N=3.

- Thus we add an adjoint chiral multiplet, φ ,with no kinetic term in the CS limit, and the matter chiral multiplets, $\Phi_i, \tilde{\Phi}_i$, which must come in pairs.

- There is a superpotential, $W = -\frac{k}{8\pi} Tr(\varphi^2)$, needed to supersymmetrize the CS term.

- Integrating out φ one obtains the same action as before, but with a superpotential:

$$W = \frac{4\pi}{k}(\tilde{\Phi}_i T^a_{R_i} \Phi_i)(\tilde{\Phi}_j T^a_{R_j} \Phi_j)$$

- These N=3 theories are completely rigid, and hence superconformal. It is impossible to have more supersymmetry in a YM-CS-matter theory, but for particular choices of gauge groups and matter representations, the pure CSM can have enhanced supersymmetry.

[Zupnik, Khetselius, Kao, Lee, Lee, Schwarz, Gaiotto, Yin]

The N=6 CSM theory of N M2 branes in C^4/Z_k

- $U(N)_k$ x $U(N)_{-k}$ CSM with a pair of bifundamental hypermultiplets

[Aharony, Bergman, Maldacena, DLJ]

- Field content:

$$A_\mu, \tilde{A}_\mu \quad \text{gauge fields}$$
$$C_I, \psi^I \text{ in } (N, \bar{N}) \quad \text{matter fields}$$
$$(C_I)^*, (\psi^I)^* \text{ in } (\bar{N}, N) \quad \text{their conjugates}$$

$$C^I = (A_a, B_{\dot{a}}^*).$$

$$W = \frac{2\pi}{k}\epsilon_{ab}\epsilon_{\dot{a}\dot{b}}(A_a B_{\dot{a}} A_b B_{\dot{b}})$$

- SU(2) x SU(2) global symmetry, which does *not* commute with $SO(3)_R$, combining to form $SU(4)_R$

[ABJM, Benna, Klebanov, Klose, Smedback, Bandres, Lipstein, Schwarz, Schanbl, Tachikawa]

‘t Hooft Limit

- The gauge theory coupling is 1/k. Fix $\lambda = N/k,\ N \to \infty$

- In the ‘t Hooft limit, one obtains IIA on $AdS_4 \times CP^3$ with N units of F_4 and k units of F_2 in CP^3

$$R_{str}^2 = 2^{5/2}\pi\sqrt{\lambda + \left\{-\frac{1}{24}\left(1-\frac{1}{k^2}\right)+\frac{\ell^2}{2k^2}\right\}}$$

$$g_{IIA} \sim \frac{\lambda^{5/2}}{N^2}$$

↰ Higher order curvature correction

[Bergman Hirano]

- For unequal ranks in the gauge theory, one obtains

$B_2 = J_{CP^3}\,((N\text{-}M)/k + ½)$. $\quad \ell = N - M - k/2$

↰ [Aharony Hashimoto Hirano Ouyang]

Monopole operators

- If we look at the theory on $S^2 \times \mathbb{R}$, in the radial quantization picture, we can imagine a configuration in which there is a single unit of magnetic flux of the overall U(1), so that $\int_{S^2} F_+ = 1$. Note that since the matter fields are neutral under this U(1), the moduli space is unaffected.

- The Chern-Simons term $k \int A_- \wedge F_+$ implies that T has (electrical) charge k under the gauge group $U(1)_-$. Equivalently, Guass' law is modified, so some matter field excitations are needed to form a gauge invariant operator. Defined $A_\pm = A_1 \pm A_2$

Moduli space

- The moduli space is determined from the F-term and D-term equations, modulo constant gauged symmetries.

$$\partial W = 0 \qquad \frac{1}{k} A_i \mu - \frac{1}{-k} \tilde{\mu} A_i = 0$$

$$\mu = A_i^\dagger A_i - B_i B_i^\dagger, \qquad \tilde{\mu} = B_i^\dagger B_i - A_i A_i^\dagger$$

- Due to the existence of monopoles, the action if only invariant under a Z_k subgroup of the constant $U(1)_-$ gauge symmetries, Λ.

$$\delta S_{CS} = \frac{k}{2\pi} \Lambda (2\pi n)$$

[Distler Mukhi Papageorgakis van Raamsdonk, ABJM]

Why is CSM a theory of M2 branes?

- There is an extra circle which emerges only at strong coupling, due to the monopole operators.
- If one gives an "eigenvalue" a VEV, $C_I = \begin{pmatrix} v & 0 \\ 0 & 0 \end{pmatrix}$, so one of the M2 branes is at distance $R = \ell_P^{3/2} v$, then the mass of the off-diagonal modes scales like

$$\frac{1}{k} v^2 = \frac{R}{k} \ell_P^{-3}$$

This is the area of a cone, rather than a length, as expected from a wrapped M2.

[Mukhi Papageorgakis, Lambert Tong, Distler Mukhi Papageorgakis van Raamsdonk, Berenstein Trancanelli]

Zoo of examples

- Consider M2 branes at the tip of an eight dimensional cone (hyperkähler, Calabi-Yau, Spin(7)) with a U(1) isometry compatible with supersymmetry. Then the reduction to IIA along that circle provides a 7d cone in which the D2 brane has a smooth near horizon geometry. In some cases there will be explicit D6 branes or orientifolds.
- Known AdS4/CFT3 dualities can deformed by marginal or relevant operators.
- Romans mass may be introduced in this way.

Singularity	IIA AdS_4 't Hooft limit	SUSY	CS-matter dual CFT
$\mathbb{C}^4$		N=8	$U(N)_1 \times U(N)_{-1}$ $U(N)_2 \times U(N)_{-2}$
$\mathbb{C}^4/\mathbb{Z}_k$	$\mathrm{AdS}_4 \times CP^3$	N=6	$U(N)_k \times U(N)_{-k}$
$\mathbb{C}^4/D_k$	$\mathrm{AdS}_4 \times CP^3/\mathbb{Z}_2$	N=5	$O(2N)_{2k} \times USp(2N)_{-k}$
$\mathbb{C}^4/\Gamma$	Quotients of CP^3	N=4	Orbifolds Necklace quivers ±k

[ABJM; Hosomichi Lee Lee Lee Park, Aharony Bergman DLJ; HLLLP, Benna Klebanov Klose Smedback, Imamura Kimura, Terashima Yagi]

Singularity	AdS_4 near horizon	SUSY	CS-matter dual CFT
HyperKähler	3-Sasakian	N=3	One loop N=3 quivers Flavor
Calabi-Yau	Sasaki Einstein	N=2	CY_3 quivers $\sum k_i = 0$ M-crystal
Spin(7) 8-manifold	Nearly G_2	N=1	Ooguri-Park Aganagic
		N=0	Armoni Naqvi Giveon Kutasov

IIA AdS_4 near horizon	SUSY	CS-matter dual CFT
Squashed and warped	N=2 N=1	Add a quadratic operator and flow
M_6 with nonzero Romans mass	N=3	$\sum k_i \neq 0$ Deformation of N=3
Massive IIA SO(4), Sp(2), SO(6) isometry	N=2,1,0	Deformation of the N=6 theory
Massive IIA	N=2	Deformation of N=2 CSM theory

Quantum corrections

- In N=3 CSM, the existence of a nonabelian R-symmetry implies that the dimensions of mesonic operators are not renormalized.
- In N=4 3d YM theories, the Yang-Mills coupling can be promoted to a component of a vector multiplet, and thus the Higgs branch (ie. moduli space of hypermultiplets) is not quantum corrected.
- This is not true for the CS coupling.

Monopoles in the chiral ring

- There are monopole operators in YM-CS-matter theories, which we follow to the IR CSM.

- In radial quantization, it is a classical background with magnetic flux $\int_{S^2} F_a = 2\pi n$, and constant scalar, $\sigma = n/2$. Of course, in the CSM limit, $\sigma_a = k^{-1}\mu$

- It is crucial that the fields in μ are not charged under a.

- This operator creates a vortex.

[Borokhov Kapustin Wu]

Anomalous dimension

- The dimension of the monopole operator will be the sum of the two contributions

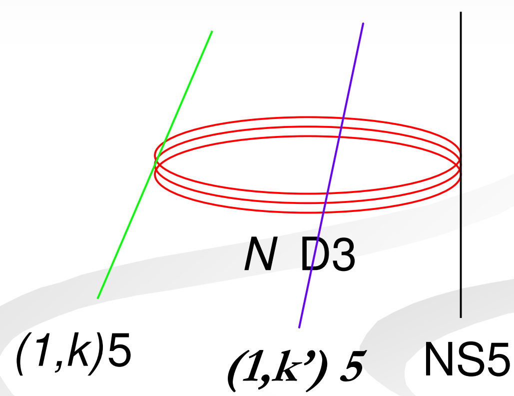
$$Q_0 = \frac{1}{2}\Big(\sum_{i\in hyper} - \sum_{i\in vector}\Big)|q_i|$$

and the dimension of the scalar fields used in the dressing. [Gaiotto Witten, Imamura, Gaiotto DLJ, Benna Klebanov Klose]

- This was calculated in Borokhov-Kapustin-Wu by quantizing the charged fermions in the monopole background with constant $\sigma = n/2$

N = 3 hypertoric 3 - Sasakian

- Wrap N D3 branes on a circle. Then N=3 supersymmetry is preserved if (p,q) 5 branes intersect the circle, and wrap 3-planes in the transverse R^6 at an angle $\theta = \arg(p\tau + q)$



- No simple description of the field theory on N D3 branes stretched between general (p,q) fivebranes is known. Restrict to the (1,k) case.

  [Kitao Ohta Ohta, Bergman Hanany Karch Kol]

Lift to M-theory geometry

- At long distances, it is natural to T-dualize, giving D2 branes in IIA. In the IR, we flow to strong coupling, lifting to M-theory. The fivebranes turn into pure geometry

$$ds^2 = U_{ij} d\vec{x}^i \cdot d\vec{x}^j + U^{ij}(d\varphi_i + A_i)(d\varphi_j + A_j)$$

[Gauntlett, Gibbons, Papadopulos, Townsend]

$$A_i = d\vec{x}^j \cdot \vec{\omega}_{ji} = dx^j_a \omega^a_{ji} \qquad \partial_{x^j_a} \omega^b_{ki} - \partial_{x^k_b} \omega^a_{ji} = \epsilon^{abc} \partial_{x^j_c} U_{ki}$$

$$U = \mathbf{1} + \sum_a \begin{pmatrix} h_a & k_a h_a \\ k_a h_a & k_a^2 h_a \end{pmatrix}, \qquad h_a = \frac{1}{2|\vec{x}_1 + k_a \vec{x}_2|}$$

- It is possible to see that there is a hyperKähler quotient singularity at the origin.

[Bielawski, Dancer]

HyperKähler cones

- There is a general relation between hypertoric hyperKähler cones and the GGPT metrics. In particular, suppose one lifts a configuration of (p, q) fivebranes. Construct the map

$$\beta = \begin{pmatrix} p_1 & p_2 & \cdots & p_n \\ q_1 & q_2 & \cdots & q_n \end{pmatrix} : U(1)^n \to U(1)^2$$

- Then the kernel defines a hyperKahler quotient of $\mathbb{H}^n$ by $U(1)^{n-2} \times Z_\ell$, which is 8 dimensional.

- For now, we only know the CFT if all q_i=1,0 .

N=3 CSM

- This field theory is just the N=3 CSM with whose quiver has one loop. The Chern-Simons levels are given by $k_i = p_i - p_{i+1}$

[Imamura Kimura; Tomasiello DLJ]

The N=2 superpotential is given by

$$W = \sum_{i=1}^{n} \frac{1}{k_i} Tr(B_i A_i - A_{i-1} B_{i-1})^2$$

The N=3 supersymmetry implies that the D term and F term equations form hyperKähler triplets

$$(k^{-1})^{ij} \not{\mu}_i T_j^{ab} C_b = 0$$

$$C_a = (A_a \ B_a^\dagger)$$

$$\mu_i^\alpha = T_i^{ab} C_a^\dagger \sigma^\alpha C_b$$

M2 branes on Calabi-Yau 4-folds

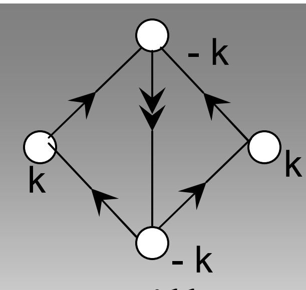
- Consider a conical Calabi-Yau 4-fold, X, with a U(1) isometry that leaves the holomorphic 4-form invariant. For example, a toric CY_4 will have 3 such U(1)'s.

- Then the Kähler quotient X//U(1) will be a CY_3, Y. Reducing M-theory on X to IIA on this U(1) gives a 7d cone which is Y fibered over a real line, with F_2 flux, varying dilaton, and $\theta_a^{FI} = k_a r$.

- The F_2 flux scales with k if one starts with X/Z_k.

[DLJ Tomasiello, Martelli Sparks, Hanany Zaffaroni, Aganagic]

Two subtleties



- If the U(1) shrinks away from the origin, then there will be explicit D6 branes in the IIA reduction. For example Q^{111}, which is a circle bundle over S^2 x S^2 x S^2, with the U(1) action generated by rotation of two of the spheres.

[work in progress of Klebanov, Pufu]

- It is also possible that Y has non-isolated orbifold singularities. For example, C^4 with a $Z_q \subset U(1)$ acting with charges 1, -1, p, -p, which is related to NS5 – (p,q)5 configurations, has a nonisolated Z_p singularity in the IIA reduction.

The weakly coupled manifestly N=4 theory is mysterious in IIB. In principle determined by Gaiotto Witten

N=2 CSM from D3 quivers

- We want to know the theory on N M2 branes on X. It is the IR limit of the theory on N D2 branes on the 7d cone X/U(1), with F_2 flux.

- Take the reduction to 2+1 of the quiver theory describing N D3 branes on Y. In the resolved geometry we can image this describes the fractional branes as D4 and D6 on holomorphic 2 and 4 cycles in Y. The CS terms arise from the D4 worldvolume coupling $\int F_2 \wedge S_{CS}(a)$

- It would be interesting to directly derive the coupling $\frac{k}{2\pi}\int D\sigma$ that must arise from the fibration of Y over R^1.

[Martelli Sparks, Hanany Zaffaroni, Hanany Vegh Zaffaroni, Niarchos, Ueda Yamazaki, Hanany He, Amariti Forcella Girardello Mariotti, Davey Hanany Mekarecya Torri, Aganagic]

Moduli space of N=2 CSM

- There are F-term equations of the usual type $\partial W = 0$. We have a D3 quiver on Y with n nodes, all fields are adjoints or bifundamentals.

- The D-term equations are replaced by cubic equations $\frac{1}{k_a}\mu_a T^i_b q_i = 0,$ where μ_a are the usual moment maps.

- The M-theory geometry is the solution $\mu_a = k_a\, r$, and the CS term implies that one only gauges the kernel of

$$\beta : (u_1, , u_n) \in U(1)^n \mapsto u_1^{k_1} ... u_n^{k_n}$$

- This is precisely the geometry X.

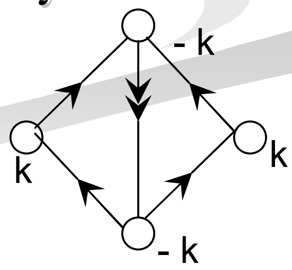
[Tomasiello DLJ, Martelli Sparks, Hanany Zaffaroni]

Toric crystals

- Descibes a toric Calabi-Yau 4-fold by reducing to IIA and T-dualizing twice on the natural T^3, then lifting back to M-theory. A stack of M2 branes becomes a stack of M5 branes wrapping T^3 and intersecting M5 branes along (p,q,r) cycles in a pattern identical to the toric diagram. [Lee, Lee Lee Park, Kim Lee Lee Park]

- Has led to some CSM proposals that don't arise as 4d quivers. Some of these are associated to M-theory U(1)'s with fixed loci, that is D6 branes in IIA.

[Franco Hanany Park Rodriguez-Gomez, Martelli Sparks,Imamura Kimura, Franco Klebanov Rodriguez-Gomez, Hanany He, Ueda Yamazaki]



Quantum effects in the nonabelian theory

- In the abelian case, the classical moduli space of these N=2 theories, which have only adjoint and bifundamental matter fields, and monopole operators with magnetic flux in the overall U(1), should be exact.

- In the nonabelian theory, one needs to compute the quantum correction to the dimension of the monopole from integrating out the charged fermions from the (off-diagonal components of) chiral and vector multiplets.

Cancellation

- Doing the calculation in the UV YM-CSM theory, $\Delta\text{dim} = \Delta R = \frac{1}{2}\sum_{fermions} \text{R-charge}$, where R is the exact R-symmetry in the IR CSM. Note that in the toric case, each chiral field appears exactly twice in the superpotential. Therefore any flavor symmetry (which must leave W invariant) that might mix with the R-symmetry cancels in the monopole dimension.

- Therefore equivalent to anomaly cancellation in the 4d YM theory with the same quiver.

- A non-trivial quantum check that the nonabelian theory is indeed the dual CFT.

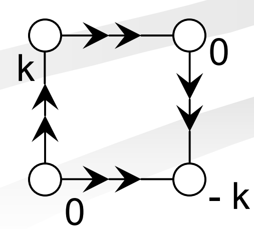
[Niarchos, Benna Klebanov Klose, work in progress]

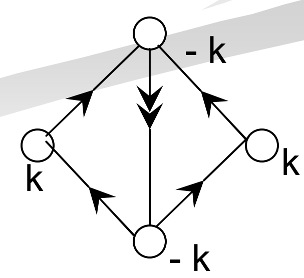
The puzzle of extra branches

- Many of these conjectured N=2 CSM duals to $AdS^4 \times SE_7$ have extra branches in the moduli space of the abelian theory. Normally such branches are associated to singular horizons, where a M2 brane can split into fractional branes. However here the meaning is mysterious, as the SE_7 can be smooth.

- Any subquiver in which the levels sum to zero may result in an extra branch, unless it is killed by F-term equations. In particular, a quiver with pure YM nodes always has Coulomb branches.

[Aganagic]





N = 1

- One idea is to engineer N=1 CSM theories by looking at N=1 4d Yang-Mills theories on a circle with domain walls that carry Chern-Simons terms. Armoni, Giveon, Israel, Niarchos recently studied the 2+1 dynamics for configurations dual to purely fractional M2 branes.

- The dual field theory to $R \times G_2$ can be obtained from F_2 flux on a CY_3, by not turning on the N=2 completion of the CS term, although it has a singular horizon in both M-theory and IIA. [Aganagic, Aganagic Vafa]

- There are proposals for the squashed S^7 with SO(5) isometry as a deformation of the N=6 CSM [Ooguri Park] and even some N=0 orientifolds [Armoni Naqvi]

RG flows

- Often a large space of classically marginal deformations, probably containing submanifolds of conformally invariant theories.

- Add a quadratic term to the N=2 superpotential. Sometimes it involves the monopole operators.

- Believed that there is a non-renormalization theorem for N=2 CSM theories, although it is not yet been proven. In particular, coefficients in the superpotential should only scale by the difference of the R-charge of the operator from 2.

[Gaiotto Yin, Niarchos]

Warped

- Adding the N=2 deformation $\Delta W = m(C_4)^a_{\hat{a}}(C^4)^b_{\hat{b}}(e^{-2\tau})^{\hat{a}\hat{b}}_{ab}$ generates the Corrado Pilch Warner flow to Warner's $SU(3) \times U(1)_R$ isometry, warped AdS_4 background. Detailed spectrum agrees.

  [Klebanov, Klose, Murugan, Klebanov, Pufu, Rocha, C. Ahn]

- Some N=1 flows were found by [Bobev Halmagyi Pilch Warner]

- There are also examples in which one flows from one purely geometric M-theory background to another. For example, the IIB configuration with an NS5 and (1,k)5 at N=2 angles and parallel in two planes engineers M2 branes on $C \times$ conifold$/Z_k$. Adding a mass term for the adjoints causes the theory to flow to C^4/Z_k.

Massive IIA

- It has long been suspected that the CFT duals to massive IIA vacua would be based on Chern-Simons, since there is a coupling $\int F_0 S_{CS}(a)$ on the D2 worldvolume. This means that the D2 theory does not flow to infinite coupling once the Romans mass is turned on. So far there is no explicitly known supersymmetric solution of a black D2 black in massive IIA on a 7d cone. The dilaton then need not vanish at the origin.

Massive IIA

- This motivates the idea that the total CS level should be related to the F_0 flux.

- Thus one considers deforming the N=6 CSM theory by the addition of a level a CS term for the second gauge group. $U(N)_k \times U(N)_{-k+a}$

- In this theory the monopole operators corresponding to D0 branes develop a tadpole, since the induced electric charge (k, a-k) cannot be cancelled with the matter fields.

[Gaiotto Tomasiello, Fujita Li Ryu Takayanagi, Petrini Zaffaroni]

- Therefore the light U(1) on the moduli space has a level *a* Chern-Simons term, matching the coupling of the D2 worldvolume to the Romans mass.

$$kCS(A_1) + (a-k)CS(A_2) + |X|^2(A_1 - A_2)^2$$

- For such deformations of N=6 CSM, there are field theories with N = 3,2,1,0 differing by the breaking of the SU(4) into flavor and R-symmetry. Still have the topology $\mathrm{AdS}_4 \times \mathbb{CP}^3$

[Tomasiello; Gaiotto Tomasiello]

Gravity duals

- For N=0 and SO(6) isometry, one must have the usual Fubini-Study metric on CP^3. There is a solution with nonzero F_0 and F_4 of this type.

- The N=1 case with Sp(2) isometry was presented at Strings 2008 by Tomasiello, and the CP^3 is described as an S^2 fibration over S^4.

- Isometry is more useful than supersymmetry, so N=2,3 solution are only known as perturbations. They would have SO(4) and SO(3) isometry if they indeed exist.

D6 branes in AdS_4

- We now know a large class of $\mathcal{N} = 3$ quiver CSM theories describing a stack of M2 branes at a hypertoric singularity. In the 't Hooft limit, the dual geometry is a warped product $\mathrm{AdS}_4 \times_w M_6$
- Introducing D6 branes wrapping an internal, homologically trivial, 3-cycle ($\mathbb{RP}^3$ in the $\mathbb{CP}^3$ case) adds fundamental hypermultiplets to the quiver. Choice of Z_2 Wilson line corresponds to which node the fundamental is attached.
- Interestingly, conformality is preserved.

[Hohenegger Kirsch, Gaiotto DLJ, Hikida Li Takayanagi, Fujita Tai]

Flavors

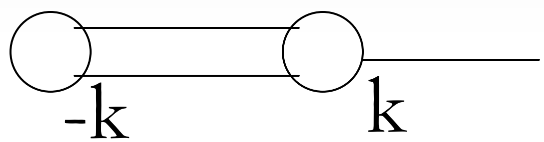
- In the IIB setup this corresponds to introducing D5 branes. The D3-D5 strings give rise to a flavor hypermultiplet in the fundamental of the node associated to that section of the wrapped D3's.

- On the "geometric" branch of the moduli space, these fields are set to zero, however they change the dimension of the monopole operators by a 1-loop correction. The metric on the moduli space is altered, resulting in $M_{quantum} = (M_{classical} \times C^4)///U(1)$ with the charge N_f $U(1)_B$ action on M and natural U(1) on C^4.

[Gaiotto DLJ, Yin DLJ]

Dual CFT for $\mathrm{AdS}_4 \times N^{010}$.

- One of the most symmetric 3-Sasakians, its cone is $T^*\mathbb{CP}^2$, and it can be written as a quotient $\frac{SU(3)}{U(1)}$



- Classical moduli space is $\mathbb{C}^4/\mathbb{Z}_k$, but for k =1 it is quantum corrected to $T^*\mathbb{CP}^2 = \mathbb{H}^3///U(1), \quad (1,1,1)$

- The SU(3) currents involve monopole operators.
- Attempts in the 90's where close:

[Billo` Fabbri Fre` Merlatti Zaffaroni, …]

Quantum corrected geometric branch

- The ordinary chiral operators are not affected.

$$Tr(A_i B_j A_k B_l)$$

- On the moduli space of diagonal matrices, the diagonal $U(1)^N$ is unbroken, and there are monopoles operators with such magnetic fluxes.

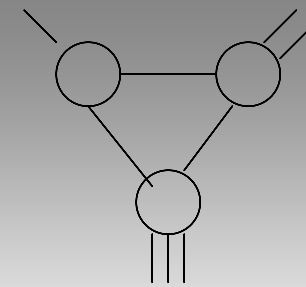
- They have CS induced charge k, and anomalous dimension $N_f/2$.

- For N_f=1, k=1, at dimension 1, one has 8 gauge invariant operators as expected for $\mathbb{H}^3////U(1)_{111}$

$$Tr(A_i B_j),\ T(B_i),\ \tilde{T}(A_i)$$

[Gaiotto DLJ]

Higgs branch



- If the number of fundamentals is at least twice the rank of the gauge groups, there is a completely Higgsed branch in which the D2 branes dissolve into the D6 branes.

- All moment maps set to zero, resulting in exactly the ordinary Kähler quotient for the ADHM quiver of N instantons of rank N_f on C^2/Z_n.

- FI parameters resolve the singularity, each node is a fractional brane that blows up into a D4.

$$k_i = \int_{S^2_{(i)}} F_2$$

Stuffing fundamentals with dof

- It is simplest to determine the number of degrees of freedom at high temperature from the M-theory supergravity limit. It is dominated by the large AdS_4 black hole, and the internal manifold only enters via the dependence of the 4d Planck scale on the volume of the 3-Sasakian.

$$\beta F = -2^{7/2} 3^{-2} \pi^2 N^{3/2} \frac{(N_f + k)\sqrt{2}}{\sqrt{N_f + 2k}} V_2 T^2 \sim \frac{N^2}{\sqrt{\lambda}} + \frac{3 N_f N}{4} \sqrt{\lambda} + \ldots$$

- Note the enhancement of $N \times N_f$!

Orientifolds

- One can place an O2 on the stack of D2 branes. These correspond in our general picture to reduction from M-theory on an S^1/Z_2 bundle.

- In the N=3 case, wrap an O3 plane around with the D3 branes. [Hosomichi Lee Lee Lee Park; Aharony Bergman DLJ; Tomasiello DLJ]

- Leads to a $O(2N)_{2k} \times USp(2N)_{-k}$ theory with N=5 supersymmetry in the CP^3 case. There is an orientifold action on the CP^3. In M-theory, there is an extra Z_2 orbifold, resulting in S^7/D_k.

Integrability

- Tremendous amount of work on this subject has built up considerable evidence for the conjecture of integrability of the U(N) × U(N) N=6 theory.

[Abbot, Agarwal, K.Ahn, Alday, Aniccto, Arutyunov, Astolfi, Bak, Beccaria, Beisert, Bombardelli, Bonelli, Bozhilov, Bykov, Caputa, Chen, d'Auria, Dukalski, Fioravanti, Fre, Frolov, Gaiotto, Gang, Giangreco, Giombi, Gomis, Grassi, Grignani, Gromov, Hamilton, Harmark, Hollowood, Kalousios, Kazakov, Kozak, Kristjansen, Jain, Lee, Lukowski, Macorini, McLoughlin, Mikhaylov, Min, Minahan, Miramontes, Murugan, Nepomechie, Nishioka, Orselli, Panigrahi, Papathanasiou, Park, Puletti, Prinsloo, Rashkov, Rey, Roiban, Ryang, Saffai, Sax, Schulgin, Semenoff, Shenderovich, Sochichiu, Sorokin, Spradlin, j. Stefanski, Strydom, Sundin, Suzuki, Takayanagi, Tsai, van Tongeren, Trigiane, Tseytlin, Uvarov, Vergu, Vieira, Volovich, Wen, Wu, Wulff, Yin, Zarembo, Zoubos, Zweibel]

Spin chain at weak ‘t Hooft coupling

- Alternating spin chain Tr(A B A B…), so only even orders contribute in the perturbative expansion in the ‘t Hooft coupling. It has SU(2|2) × U(1) symmetry.

- 2-loop dilatation operator was shown to be integrable in the scalar sector.

[Minahan Zarembo, Bak Rey, …]

- Full OSp(6|4) dilatation operator and Bethe ansatz was proposed, checked extensively at 2 loops, as well as some 4 loop results.

[Bak Min Rey]

[Gaiotto Giombi Yin, Grignani Harmark Orselli, Gromov Vieira, Ahn Nepomechie, Sundin, Zweibel, Minahan Schulgin Zarembo, …]

IIA string dynamics at strong 't Hooft coupling

[Arutymov Frolov, j. Stefanski, Bak Rey, Gromov Vieira, Gomis Sorokin Wulff, Alday Arutymov Bykov, …]

- Found to be integrable to leading order in $1/\sqrt{\lambda}$.

- The full IIA string worldsheet is not quite a coset model, although it contains the integrable $OSp(6|4)/U(3) \times SO(1,3)$ as a subsector .

  [Arutymov Frolov, j. Stefanski, Fre Grassi, d'Auria Fre Grassi Trigiante, …]

- The 11 dimensional $OSp(8|4)/SO(7) \times SO(1,3)$ is Hopf fibered over the 10 d superspace model with 32 odd directions and 24 supersymmetries.

  [Gomis Sorokin Wulff]

Dispersion relation

- Gaint magnon dispersion relation $\epsilon = \sqrt{1 + h(\lambda)\sin^2\frac{p}{2}}$ where p is the momentum can be calculated both using integrability [Gaiotto Giombi Yin, Grignani Harmark Orselli] or by treating off-diagonal modes perturbatively [Berenstein Trancanelli, Trancanelli]

- It would be interesting to determine the interpolating function appearing in the dispersion relation exactly.

Condensed matter applications

- Often involve 2+1 dimensional field theories.
- Chern-Simons terms are very common.
- Hope is to find some semi-qualitative matching between such theories and supersymmetric ones.
- However, gauge groups are usually abelian - although SU(2) seems to be possible.
- Can at least provide string theory realizations, if not gravity duals.

Fractional quantum Hall effect

- Two dimensional electron system with an abelian Chern-Simons term at CS equal to the inverse of the filling fraction.

- Engineered by adding a flavor D6 brane, coupling to a background electric field in the AdS4, $C_3 = \frac{4\pi k}{R^3} A_{ext} \wedge \omega_{CP3}$ and turning on M units of $B_{2.}$ An abelain CS term is induced for the 2+1 dual to A_{ext}, whose level can be dialed by rotating the D6 brane.

[Fujita, Hikida, Li, Ryu, Takayanagi; Alanen, Keski-Vakkuri, Kraus, Suur-Uski]

Superconducting M2 branes

- Charged operators condense below a critical temperature for many N=2 Chern-Simons-matter theories at nonzero chemical potential.
- Turn on the external electric field in AdS4. There can be charged pseudoscalars coming from particular internal C_3. [Denef Hartnoll]
- A supersymmetric Schrödinger invariant limit of the mass deformed theory [Hosomichi Lee Lee Lee Park, Gomis Rodriguez-Gomez van Raamsdonk Verlinde] has also been studied.

[Nakayama, Lee, Lee, Lee, Kawai, Rey, Ryu, Sakaguchi, Sasaki, Yoshida]

Summary

- Reviewed the explicit Lagrangian description of the IR limit of nonabelian M2 branes.
- New progress in extending AdS4/CFT3 duality to examples with less supersymmetry, fluxes, flavor D-branes, Romans mass…
- Better understanding of monopole operators and quantum corrections.
- Advertised developments in integrability and condensed matter applications.

Future directions

- Complete the analysis of N=2 CSM duals to M2 branes on CY_4. Explore N=1 theories, for example in the large N_f limit.
- Duals for landscape IIA vacua.
- Learn about the strong coupling limit of massive IIA.
- Explain $N^{3/2}$ - derive the $1/\sqrt{\lambda}$ suppression of dofs.
- More tests of integrability.
- Further explore the connection to 2+1 condensed matter systems.