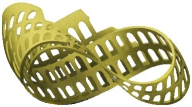
# Stringy Instantons and Duality

Alberto Lerda

U.P.O. *"A. Avogadro"* & I.N.F.N. - Alessandria

Romæ, a.d. VIII Kal. Iul. anno MMDCCLXII a.U.c.

Roma, June 24, 2009



# Foreword

- This talk is mainly based on:
  - M. Billò, M. Frau, L. Gallot, A. L. and I. Pesando, JHEP **0903** (2009) 056 , arXiv:0901.1666 [hep-th]
  - M. Billò, L. Ferro, M. Frau, L. Gallot, A. L. and I. Pesando, arXiv:0905.4586 [hep-th]
  - M. Billò, M. Frau, F. Fucito, A.L., J.F. Morales, R. Poghossian, work in progress
- Very extensive (± recent) literature; for a review see *e.g.*
  - R. Blumenhagen, M. Cvetic, S. Kachru and T. Weigand, arXiv:0902.3251 [hep-th].

# Plan of the talk

1. Introduction and motivations
2. Gauge vs. stringy instantons in D-brane models
3. The D7/D(–1) system in type I′
   - Non-perturbative couplings
   - Duality with heterotic string
4. Conclusions and perspectives

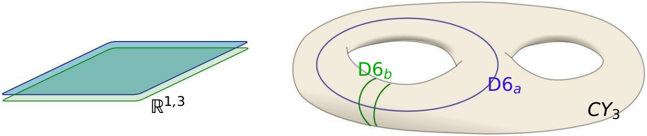
# Introduction and motivations

- The possibility of acquiring control over non-perturbative effects has been a unifying theme behind many developments in string theory.
- Recently, there has been a growing interest in the effects induced by D-instantons or, more generally, by Euclidean D-branes (E-branes):
  - They allow to reproduce the (standard) instanton calculus in string theory
    Polchinski, 1994; ...; Green+Gutperle, 2000; ...; Billò et al. 2002; ...
  - They may give rise to non-perturbative couplings that are forbidden in perturbation theory but necessary for phenomenological applications (neutrino masses, Yukawa couplings, ...)
    Blumenhagen et al, 2006; Ibanez + Uranga, 2006; ...
- Like instantons in gauge theories, also the instantonic branes lead to a particularly tractable class of non-perturbative phenomena in string models.

# Introduction and motivations

- In string theory $4d$ supersymmetric gauge theories are engineered with systems of space-filling D-branes, *i.e.* D-branes that entirely fill the $4d$ space-time and are (partially or totally) wrapped in the internal $CY_3$ space.

  For example, in Type IIA one takes (intersecting) D6 branes wrapping 3-cycles in $CY_3$.



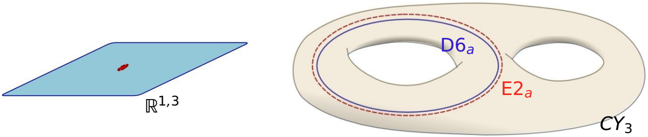

(artwork by Marco Billò)

# Introduction and motivations

- In these brane-world models, instantons are engineered with Euclidean branes, *i.e.* D-branes that are point-like in the 4$d$ space-time and are totally wrapped in the internal $CY_3$ space.

  For example, in Type IIA we have E2 branes wrapping 3-cycles in $CY_3$.

- First possibility:
  - The gauge and the instantonic branes wrap the SAME cycle




  - These are the usual gauge instantons.

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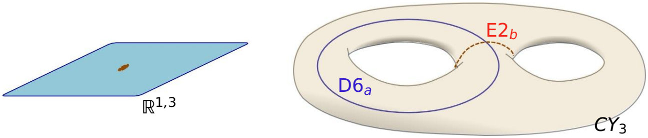
- Second possibility:
  - The gauge and the instantonic branes wrap the DIFFERENT cycles




  - These are the so-called exotic or stringy instantons.

# Introduction and motivations

- Instanton effects in string theory have been studied over the years from various standpoints, mainly exploiting string duality
- Recently concrete tools have been developed to directly compute non-perturbative effects using perturbative string theory methods for ordinary gauge instantons

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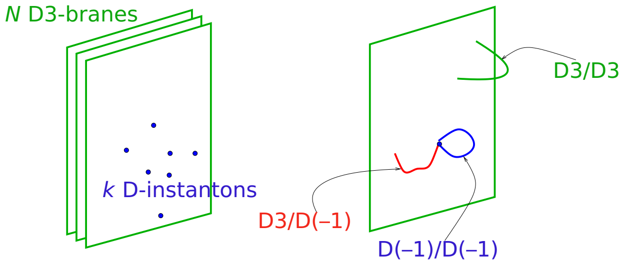
- Instanton effects in string theory have been studied over the years from various standpoints, mainly exploiting string duality
- Recently concrete tools have been developed to directly compute non-perturbative effects using perturbative string theory methods for ordinary gauge instantons
- I will discuss if and how these methods extend to the exotic or stringy instantons and consider a particular example in some detail

# Gauge vs. Stringy Instantons

- Instantonic branes that are equal to the space-filling branes in the internal space, represent ordinary gauge instantons

| | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $Z^1$ | $Z^2$ | $Z^3$ | $Z^4$ | $Z^5$ | $Z^6$ |
|---|---|---|---|---|---|---|---|---|---|---|
| D3 | − | − | − | − | * | * | * | * | * | * |
| D(−1) | * | * | * | * | * | * | * | * | * | * |

$\Rightarrow$ $Z^i$ untwisted

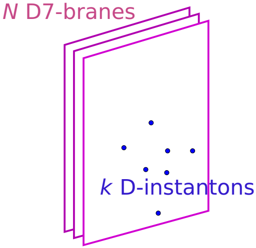


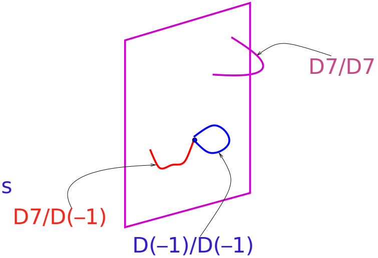
# Gauge vs. Stringy Instantons

- Instantonic branes that are different from the space-filling branes in the internal space, represent exotic instantons

| | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $Z^1$ | $Z^2$ | $Z^3$ | $Z^4$ | $Z^5$ | $Z^6$ |
|---|---|---|---|---|---|---|---|---|---|---|
| D7 | − | − | − | − | − | − | − | − | * | * |
| D(−1) | * | * | * | * | * | * | * | * | * | * |

$\Rightarrow$ $Z^{1,\ldots,4}$ mixed



# Gauge vs. Stringy Instantons

- The open strings ending on the D(–1)'s describe the instanton moduli. Consider in particular the strings stretching between the gauge and the instantonic branes:
- In the NS sector the physicity condition is

$$L_0 = N_X + N_\Psi + \frac{1}{2}\sum_i \theta_i = 0 \qquad (N_X, N_\Psi, \theta_i \geq 0)$$

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Gauge instantons:

- $\theta_i = 0$
- There are bosonic ADHM moduli $\omega_{\dot\alpha}$ related to the instanton size $\rho$.

Stringy instantons:

- $\theta_i \neq 0$
- There are "more than 4" ND directions and no size moduli, like $w_{\dot\alpha}$, exist.

# Gauge vs. Stringy Instantons

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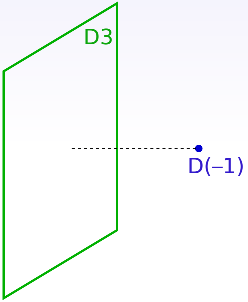

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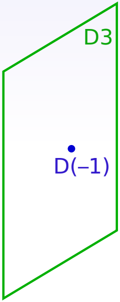

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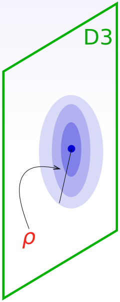


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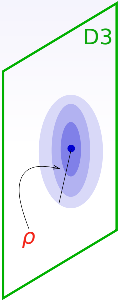


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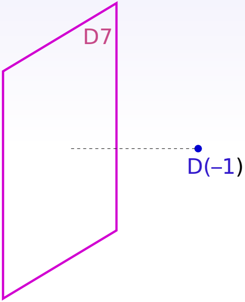

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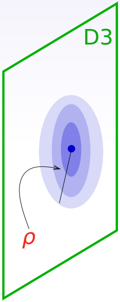


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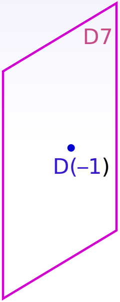


The stringy instantons are point-like configurations

# Gauge vs. Stringy Instantons

- There are other important differences in the fermionic moduli:
- In the R sector the physicity condition is

$$L_0 = N_X + N_\Psi = 0 \qquad \text{(No } \theta_i\text{-dependence!)}$$

and there are always massless modes!

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Gauge instantons:

- Chiral moduli $M^\alpha \sim \theta^\alpha$
- Anti-chiral moduli $\lambda_{\dot\alpha}$
- The $\lambda_{\dot\alpha}$'s interact and are Lagrange multipliers for the fermionic ADHM constraints

Stringy instantons:

- Chiral moduli $M^\alpha \sim \theta^\alpha$
- Anti-chiral moduli $\lambda_{\dot\alpha}$
- The $\lambda_{\dot\alpha}$'s do not interact and are true zero-modes: they do not appear in the moduli action!

- The contribution of fermionic moduli is different in the two cases

# Gauge vs. Stringy Instantons

- In stringy instantons, one has to remove or lift the anti-chiral zero-modes $\lambda_{\dot{\alpha}}$ in order to have non-vanishing results:
- Various possibilities:
  - orientifold projection

    Argurio et al, 2007; Bianchi et al, 2007; ...
  - closed string fluxes

    Blumenhagen et al, 2007; ... ; Billò et al, 2008; Uranga, 2008; ...
  - adding extra branes

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    Petersson, 2007; Ferretti et al, 2009; ...
- I will consider this problem in a simpified setting and study the

  **D7/D(–1) system in type I′**

  which, in many respects, provides the prototypical example of stringy instantons.

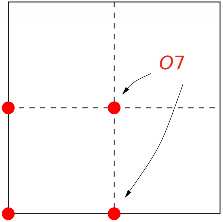
# The D7/D(–1) system in type I′

- Type I′ is type IIB on a 2-torus $T_2$ modded out by

$$\Omega = \omega\,(-1)^{F_L}\,\mathcal{I}_2$$

where $\omega$= world-sheet parity, $F_L$ = left-moving fermion number, $\mathcal{I}_2$ = inversion on $T_2$

- $\Omega$ has four fixed-points on $T_2$ where four O7-planes are placed
- It admits D(–1), D3 and D7-branes transverse to $T_2$



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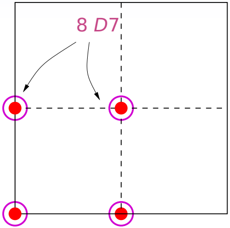
- Local RR tadpole cancellation requires eight D7-branes at each fix point



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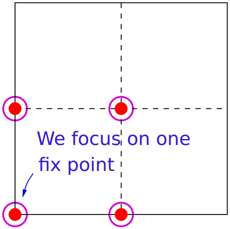

# The D7/D7 sector

- On the D7's, we have a 8*d* gauge theory with group SO(8) whose action is

$$
\begin{aligned}
S &= S_{(2)} + S_{(4)} + S_{(5)} + \cdots \\
&= \frac{1}{8\pi g_s}\int d^8x\,\Big[\frac{\mathrm{Tr}\big(F^2\big)}{(2\pi)^4\alpha'^2} - \frac{t_8\,\mathrm{Tr}\big(F^4\big)}{3\,(2\pi)^2} + \alpha'\mathcal{L}_{(5)}\big(F,DF\big) + \cdots\Big]
\end{aligned}
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- The quadratic Yang-Mills term $S_{(2)}$ has a dimensionful coupling $g^2_{\mathrm{YM}} \equiv 4\pi g_s(2\pi\sqrt{\alpha'})^4$

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- Contributions of higher order in $\alpha'$

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\end{aligned}
$$

- The quartic term has a dimensionless coupling:

$$
S_{(4)} = -\frac{1}{96\pi^3 g_s} \int d^8x\, t_8 \mathrm{Tr}\left(F^4\right)
$$

# The D7/D7 sector

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- Adding the WZ term, we can write

$$S_{(4)} = -\frac{1}{4!\,4\pi^3 g_s} \int d^8x\, t_8 \mathrm{Tr}\left(F^4\right) - 2\pi \mathrm{i}\, C_0\, c_{(4)}$$

where $c_{(4)}$ is the fourth Chern number

$$c_{(4)} = \frac{1}{4!(2\pi)^4} \int \mathrm{Tr}\left(F \wedge F \wedge F \wedge F\right)$$

# The D7/D7 sector

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- Adding the fermionic terms, we can write

$$S_{(4)} = \frac{1}{(2\pi)^4} \int d^8x\, d^8\theta\ \mathrm{Tr}\left[\frac{\mathrm{i}\pi}{12}\,\tau\,\Phi^4\right] + \text{ c.c.}$$

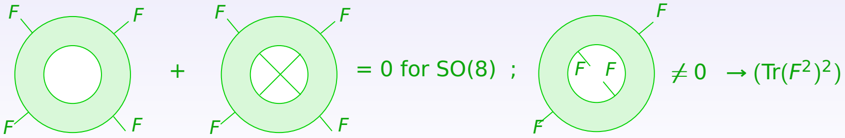
where

$$\tau = C_0 + \frac{\mathrm{i}}{g_s} \qquad \text{(axion-dilaton)}$$

$$\Phi(x,\theta) = \varphi(x) + \theta\Lambda + \frac{1}{2}\theta\gamma^{\mu\nu}\theta F_{\mu\nu} + \ldots \qquad (\mathcal{N} = 2 \text{ vector superfield})$$

# The D7/D7 sector: 1-loop

- At 1-loop we have




The corresponding quartic term is given by

$$S_{(4)}^{1-\text{loop}} = \frac{1}{256\pi^4} \int d^8x \log\left(\text{Im}\tau\, \text{Im}U\, |\eta(U)|^4\right) t_8 \left(\text{Tr}F^2\right)^2 \Big]$$
$$= \frac{1}{(2\pi)^4} \int d^8x\, d^8\theta\, \left[\frac{1}{32} \log\left(\text{Im}\tau\, \text{Im}U\, |\eta(U)|^4\right) \left(\text{Tr}\Phi^2\right)^2\right] + \text{c.c.}$$

where $U$ is the complex structure of the 2-torus $T_2$ and $\eta(U)$ the Dedekind function.

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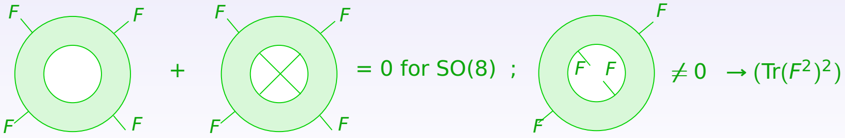


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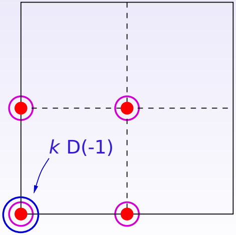
where $U$ is the complex structure of the 2-torus $T_2$ and $\eta(U)$ the Dedekind function.

- What about the NON-PERTURBATIVE contributions?

# Adding D-instantons




- Add $k$ D-instantons on top of the 8 D7's.

- Classical action of the D(-1)'s is

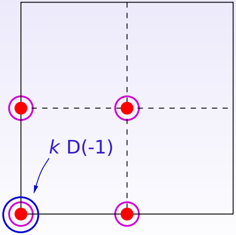
$$S_{\rm cl} = -2\pi \mathrm{i}\, k\tau$$

- It coincides with the quartic action $S_{(4)}$ when $c_{(4)} = k$:

$$c_{(4)} = \frac{1}{4!(2\pi)^4} \int \mathrm{Tr}\Big(F \wedge F \wedge F \wedge F\Big) = k \quad \Rightarrow$$

$$\Rightarrow \int d^8x\, t_8\, \mathrm{Tr}\Big(F^4\Big) = -\frac{4!}{2}(2\pi)^4\, c_{(4)} \quad \Rightarrow \quad \boxed{S_{(4)} = -2\pi \mathrm{i}\, k\tau = S_{\rm cl}}$$

# Adding D-instantons




- Add $k$ D-instantons on top of the 8 D7's.

  - Classical action of the D(-1)'s is

$$S_{\rm cl} = -2\pi \mathrm{i}\, k \tau$$

  - Strict analogy with $4d$ self-dual YM configurations in D3/D(-1) systems for which $c_{(2)} = k$.

  - It suggests a relation to some $8d$ "instanton" solution of the quartic action

Billò et al; 2009

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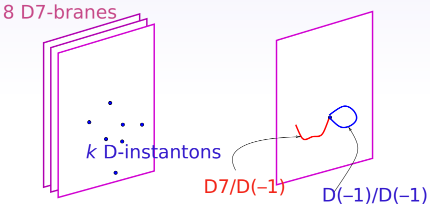
# The moduli spectrum in the D7/D(–1) system




- Open strings with at least one end on a D(–1) carry no momentum: they are moduli rather than dynamical fields.
- There are "more than four" mixed ND directions → exotic instantons

# The moduli spectrum in the D7/D(–1) system

| Sector | Name | Meaning | Chan-Paton | Dimension |
|---|---|---|---|---|
| D(–1)/D(–1) (NS) | $a_\mu$ | *centers* | symm SO($k$) | (length) |
| | $\chi$ , $\bar{\chi}$ | | adj. SO($k$) | $(\text{length})^{-1}$ |
| | $D_m$ | *auxiliary* | adj. SO($k$) | $(\text{length})^{-2}$ |
| D(–1)/D(–1) (R) | $M^\alpha$ | *partners* | symm SO($k$) | $(\text{length})^{1/2}$ |
| | $\lambda_{\dot\alpha}$ | | adj. SO($k$) | $(\text{length})^{-3/2}$ |
| D7/D(–1) (R) | $\mu$ | | $\mathbf{8} \times \mathbf{k}$ | $(\text{length})^{1/2}$ |
| D7/D(–1) (NS) | $w$ | *auxiliary* | $\mathbf{8} \times \mathbf{k}$ | $(\text{length})^{0}$ |

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| D(–1)/D(–1) (R) | $M^\alpha$ | *partners* | symm SO($k$) | $(\text{length})^{1/2}$ |
| | $\lambda_{\dot{\alpha}}$ | | adj. SO($k$) | $(\text{length})^{-3/2}$ |
| D7/D(–1) (R) | $\mu$ | | $\mathbf{8} \times \mathbf{k}$ | $(\text{length})^{1/2}$ |
| D7/D(–1) (NS) | $w$ | *auxiliary* | $\mathbf{8} \times \mathbf{k}$ | $(\text{length})^{0}$ |

- The SO($k$) representation is determined by the orientifold projection

# The moduli spectrum in the D7/D(–1) system

| Sector | Name | Meaning | Chan-Paton | Dimension |
|---|---|---|---|---|
| D(–1)/D(–1) (NS) | $a_\mu$ | *centers* | symm SO($k$) | (length) |
| | $\chi$ , $\bar{\chi}$ | | adj. SO($k$) | $(\text{length})^{-1}$ |
| | $D_m$ | *auxiliary* | adj. SO($k$) | $(\text{length})^{-2}$ |
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| D7/D(–1) (NS) | $w$ | *auxiliary* | $\mathbf{8} \times \mathbf{k}$ | $(\text{length})^{0}$ |

- The abelian part of $\lambda_{\dot\alpha}$ is a dangerous zero-mode but orientifold projection removes it

# The moduli spectrum in the D7/D(–1) system

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|---|---|---|---|---|
| D(–1)/D(–1) (NS) | $a_\mu$ | *centers* | symm SO($k$) | (length) |
| | $\chi$ , $\bar{\chi}$ | | adj. SO($k$) | (length)$^{-1}$ |
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| D7/D(–1) (NS) | $w$ | *auxiliary* | $\mathbf{8} \times \mathbf{k}$ | (length)$^0$ |

- The mixed sector only contains fermionic physical moduli (the only bosons are the dimensionless auxiliary variables $w$)
- This is a typical feature of the “stringy” instantons

# The moduli action

The interactions among instanton moduli, collectively denoted as $\mathcal{M}_{(k)}$, are described by

$$\mathcal{S} = \mathcal{S}_1 + \mathcal{S}_2$$

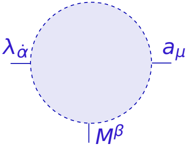
where

$$\mathcal{S}_1 = \mathrm{tr}\Big\{\mathrm{i}\,\lambda_{\dot\alpha}\gamma_\mu^{\dot\alpha\beta}[a^\mu, M_\beta] + \frac{1}{2g_0^2}\,\lambda_{\dot\alpha}[\chi,\lambda^{\dot\alpha}] + M^\alpha[\bar\chi, M_\alpha]$$
$$+\frac{1}{2g_0^2}D_m D^m - \frac{1}{2}D_m(\tau^m)_{\mu\nu}\,[a^\mu, a^\nu] + \big[a_\mu,\bar\chi\big]\,[a^\mu,\chi] + \frac{1}{2g_0^2}\,[\bar\chi,\chi]^2\Big\}$$

$$\mathcal{S}_2 = \mathrm{tr}\Big\{{}^{\mathrm{t}}\mu\,\mu\,\chi\Big\} + \mathrm{tr}\Big\{{}^{\mathrm{t}}\mu\,\Phi(x,\theta)\,\mu\Big\} + \mathrm{tr}\Big\{{}^{\mathrm{t}}w\,w\Big\}\quad , \qquad g_0^2 = \frac{g_s}{4\pi^3\alpha'^2}$$

# The moduli action

$$\mathcal{S}_1 = \mathrm{tr}\Big\{ \mathrm{i}\,\lambda_{\dot\alpha}\gamma_\mu^{\dot\alpha\beta}[a^\mu, M_\beta] + \frac{1}{2g_0^2}\,\lambda_{\dot\alpha}[\chi, \lambda^{\dot\alpha}] + M^\alpha[\bar\chi, M_\alpha]$$
$$+ \frac{1}{2g_0^2} D_m D^m - \frac{1}{2} D_m (\tau^m)_{\mu\nu}\,[a^\mu, a^\nu] + \left[a_\mu, \bar\chi\right][a^\mu, \chi] + \frac{1}{2g_0^2}\,[\bar\chi, \chi]^2 \Big\}$$



# The moduli action

$$\mathcal{S}_1 = \mathrm{tr}\Big\{ \mathrm{i}\,\lambda_{\dot\alpha}\gamma_\mu^{\dot\alpha\beta}[a^\mu, M_\beta] + \frac{1}{2g_0^2}\lambda_{\dot\alpha}[\chi,\lambda^{\dot\alpha}] + M^\alpha[\bar\chi, M_\alpha]$$
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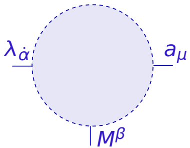


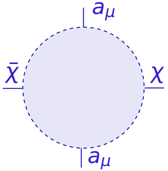

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$$
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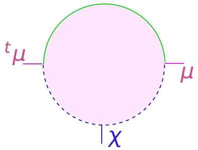

# The moduli action

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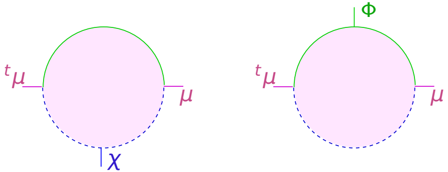

# Effective action from D-instantons

- The total instanton action is

$$\mathcal{S}_{\text{inst}} = -2\pi \mathrm{i}\tau k + \mathcal{S}_{(1)} + \mathcal{S}_{(2)} \equiv -2\pi \mathrm{i}\tau k + \mathcal{S}(\mathcal{M}_{(k)}, \Phi)$$

- The non-perturbative contributions to the effective action of the gauge degrees of freedom $\Phi$ are obtained by integrating over the D-instanton moduli $\mathcal{M}_{(k)} = \{x, \theta, \widehat{\mathcal{M}}_{(k)}\}$ and summing over all instanton numbers $k$

$$\mathcal{S}_{\text{n.p.}}(\Phi) = \sum_k e^{2\pi \mathrm{i}\tau k} \int d\mathcal{M}_{(k)}\, e^{-\mathcal{S}(\mathcal{M}_{(k)}, \Phi)}$$

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$$\mathcal{S}_{\text{n.p.}}(\Phi) = \int d^8x\, d^8\theta \sum_k q^k \int d\widehat{\mathcal{M}}_{(k)}\, e^{-\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \Phi(x,\theta))}$$

with

$$q = e^{2\pi \mathrm{i}\tau}$$

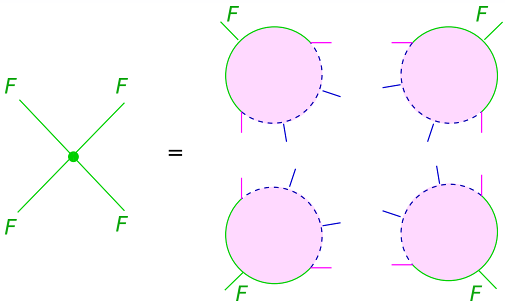
# Effective action from D-instantons

- The scaling dimensions of the centered moduli imply that

$$\left[d\widehat{\mathcal{M}}_{(k)}\right] = (\text{length})^{-4}$$

- Thus $$\int d\widehat{\mathcal{M}}_{(k)}\, e^{-\mathcal{S}(\widehat{\mathcal{M}}_{(k)},\Phi(x,\theta))} = \text{quartic invariant in } \Phi(x,\theta)$$

- Integration over $d^8\theta$ leads to quartic terms of the form " $t_8\,F^4$ "



# One-instanton: $k = 1$

- For $k = 1$ things are particularly simple
  - The spectrum of moduli is reduced to $\{x, \theta, \mu\}$
  - The moduli action is simply $\mathcal{S}_{\text{inst}} = -2\pi \mathrm{i}\tau + {}^{t}\mu\, \Phi(x,\theta)\, \mu$
- The instanton-induced interactions are thus

$$\int d^8x\, d^8\theta\; q \int d\mu\, e^{-{}^{t}\mu\, \Phi(x,\theta)\, \mu} \sim \int d^8x\, d^8\theta\; \boxed{q\, \mathrm{Pf}\big(\Phi(x,\theta)\big)}$$

- A new structure, associated to the SO(8) invariant " $t_8 \mathrm{Pf}(F)$ ", appears in the effective action at the 1-instanton level after the $d^8\theta$ integration

# Multi-instantons: $k > 1$

- For $k > 1$ things are more complicated, but ...
- Substantial progress can be made by exploiting the SUSY properties of the moduli action, which lead to:
  - an equivariant cohomological BRST structure
  - a localization of the moduli integrals (after suitable closed string deformations)
- Similar techniques have been successfully used to
  - compute the YM integrals in $d = 10, 6, 4$ and the D-instanton partition function
    Moore+Nekrasov+Shatashvili, 1998
  - compute multi-instanton effects in $\mathcal{N} = 2$ SYM in $d = 4$ and compare with the Seiberg-Witten solution
    Nekrasov, 2002; + ...
  - derive the multi-instanton calculus using D3/D(–1) brane systems
    Fucito et al, 2004; Billò et al, 2006; ...

# BRST structure

- Choose one of the SUSY charges $Q_{\dot\alpha}$ (preserved by both the D7's and the D(–1)'s), say $Q_{\dot 8} \equiv Q$

- After relabeling $M_\alpha \to M_\mu \equiv (M_m, -M_8)$ and $\lambda_{\dot\alpha} \to (\lambda_m, \eta)$ one has

$$Qa = M \quad , \quad QM = [\chi, a] \quad , \quad Q\lambda = D \quad , \quad QD = [\chi, \lambda] \quad ,$$
$$Q\bar\chi = \eta \quad , \quad Q\eta = [\chi, \bar\chi] \quad , \quad Q\mu = w \quad , \quad Qw = \chi\,\mu + \mu\,\varphi \quad ,$$
$$Q\chi = 0 \quad .$$

- The complete moduli action is BRST exact:

$$\boxed{\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi) = Q\,\Psi} \qquad\qquad \varphi = \langle \Phi(x, \theta) \rangle$$

# BRST structure

- On any modulus we have

$$Q^2 \bullet = \left[\, T_{\mathrm{SO}(k)}(\chi) + T_{\mathrm{SO}(8)}(\varphi) \,\right] \bullet$$

  where

  - $T_{\mathrm{SO}(k)}(\chi) =$ infinitesimal rotation of SO($k$) parametrized by $\chi$
  - $T_{\mathrm{SO}(8)}(\varphi) =$ infinitesimal rotation of SO(8) parametrized by $\varphi$

# BRST structure

- On any modulus we have

$$Q^2 \bullet = \left[ \, T_{SO(k)}(\chi) + T_{SO(8)}(\varphi) \, \right] \bullet$$

- The moduli action $\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi)$ is invariant also under an auxiliary SO(7) group

| | SO($k$) | SO(8) | SO(7) |
|---|---|---|---|
| $a^\mu$ , $M^\mu$ | symm | **1** | $\mathbf{8}_s$ |
| $D_m$ , $\lambda_m$ | adj | **1** | **7** |
| $\chi$ , $\bar{\chi}$ , $\eta$ | adj | **1** | **1** |
| $\mu$ , $w$ | **k** | $\mathbf{8}_v$ | **1** |
| $\varphi$ | **1** | adj | **1** |

# BRST structure

- On any modulus we have

$$Q^2 \bullet = \left[\ T_{SO(k)}(\chi) + T_{SO(8)}(\varphi)\ \right] \bullet$$

- The moduli action $\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi)$ is invariant also under an auxiliary SO(7) group

- We could replace $Q$ with a modified BRST charge $\widetilde{Q}$, nilpotent also up to SO(7) transformations:

$$\widetilde{Q}^2 \bullet = \left[\ T_{SO(k)}(\chi) + T_{SO(8)}(\varphi) + T_{SO(7)}(\mathcal{F})\ \right] \bullet$$

# Deformed BRST structure and RR background

- Such a deformation allows to

  regulate and localize

  the integrals on the instanton moduli

  Moore+Nekrasov+Shatshvili, 1998; Nekrasov, 2002;

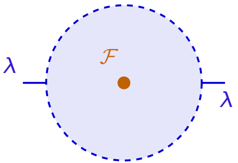
- This deformation can be obtained by putting the D7/D(–1) in a closed string RR background ("graviphoton"-like):

  Billò et al, 2009

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu z} \quad , \quad \bar{\mathcal{F}}_{\mu\nu} \equiv \bar{F}_{\mu\nu\bar{z}}$$

*i.e.* a RR 3-form field strength with one index on the 2-torus $T_2$. Insertion of RR vertex operators

in disc diagrams modify the moduli action




$$\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi) \to \mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi, \mathcal{F})$$

# Deformed BRST structure and RR background

- In presence of this RR background, the moduli action remains BRST-exact:

$$\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi, \mathcal{F}) = \widetilde{Q}\,\widetilde{\Psi}$$

where, for example,

$$\widetilde{Q}M^{\mu} = [\chi, a^{\mu}] - \frac{1}{2}\mathcal{F}^{\mu\nu}a_{\nu}$$

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where, for example,

$$\widetilde{Q}M^{\mu} = [\chi, a^{\mu}] - \frac{1}{2}\mathcal{F}^{\mu\nu}a_{\nu}$$

- This structure allows to suitably rescale the integration variables and show that

the semiclassical approximation is exact

Moore+Nekrasov+Shatashvili, 1998; ...; Nekrasov, 2002; Flume+Poghossian, 2002; Bruzzo et al, 2003; ...

$$Z_k(\varphi, \mathcal{F}) \equiv \int d\mathcal{M}_{(k)}\; e^{-\mathcal{S}(\widehat{\mathcal{M}}_{(k)}, \varphi, \mathcal{F})} \;=\; \ldots \;=\; \ldots$$

$$= \int \{da\, dM\, dD\, d\lambda\, d\mu\, d\chi\}\; e^{-\mathrm{tr}\{\frac{g}{2}D^2 - \frac{g}{2}\lambda\widetilde{Q}^2\lambda + \frac{t}{4}a\widetilde{Q}^2 a + \frac{t}{4}M^2 + {}^{t}\mu\widetilde{Q}^2\mu\}}$$

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$$Z_k(\varphi, \mathcal{F}) = \int\{da\,dM\,dD\,d\lambda\,d\mu\,d\chi\}\; e^{-\mathrm{tr}\{\frac{g}{2}D^2 - \frac{g}{2}\lambda\widetilde{Q}^2\lambda + \frac{t}{4}a\widetilde{Q}^2a + \frac{t}{4}M^2 + {}^t\mu\widetilde{Q}^2\mu\}}$$

$$\sim \int\{d\chi\}\;\frac{\mathrm{Pf}_{\lambda}\left(\widetilde{Q}^2\right)\,\mathrm{Pf}_{\mu}\left(\widetilde{Q}^2\right)}{\det_{a}\left(\widetilde{Q}^2\right)^{1/2}}$$

# Explicit results

- The last step can be done as a contour integral like in Moore + Nekrasov + Shatashvili (hep-th/9803265) and the final result for $Z_k(\varphi, \mathcal{F})$ comes from a sum over residues
- From the explicit expression of $Z_k(\varphi, \mathcal{F})$, we can obtain the non-perturbative effective action. But there are two caveats:

  1) At instanton number $k$, there are disconnected contributions from smaller instantons $k_i$ (with $\sum_i k_i = k$). To isolate the connected components we have to take the log

  $$\mathcal{Z} = \sum_k Z_k(\varphi, \mathcal{F})\, q^k \;\longrightarrow\; \log \mathcal{Z}$$

  2) In obtaining $Z_k(\varphi, \mathcal{F})$ we integrated also over $x$ and $\theta$ producing a factor of $\mathcal{E}^{-1} \sim \det(\mathcal{F})^{-1/2}$. To remove this contribution we have to multiply by $\mathcal{E}$

  $$\log \mathcal{Z} \;\longrightarrow\; \mathcal{E} \log \mathcal{Z}$$

  before turnig-off the RR deformation.

# Explicit results

- The last step can be done as a contour integral like in Moore + Nekrasov + Shatashvili (hep-th/9803265) and the final result for $Z_k(\varphi, \mathcal{F})$ comes from a sum over residues
- Taking all this into account, we obtain the non-perturbative part of the D7-brane effective action:

$$S^{(\mathrm{n.p.})} = \frac{1}{(2\pi)^4} \int d^8x\, d^8\theta\, F^{(\mathrm{n.p.})}\big(\Phi(x,\theta)\big)$$

with the "prepotential" $F^{(\mathrm{n.p.})}(\Phi)$ given by

$$\boxed{F^{(\mathrm{n.p.})}(\Phi) = \mathcal{E} \log \mathcal{Z}\Big|_{\varphi\to\Phi,\mathcal{F}\to 0}}$$

with

$$\mathcal{Z} = \sum_k Z_k(\varphi, \mathcal{F})\, q^k \qquad , \qquad \mathcal{E} \sim \det(\mathcal{F})^{1/2}$$

# Explicit results

- Expanding in instanton numbers, $F^{(\text{n.p.})} = \sum_k q^k F_k$, we have

$$F_1 = \mathcal{E} Z_1 \ ,$$

$$F_2 = \mathcal{E} Z_2 - \frac{F_1^2}{2\mathcal{E}} \ ,$$

$$F_3 = \mathcal{E} Z_3 - \frac{F_2 F_1}{\mathcal{E}} - \frac{F_1^3}{6\mathcal{E}^2}$$

$$F_4 = \mathcal{E} Z_4 - \frac{F_3 F_1}{\mathcal{E}} - \frac{F_2^2}{2\mathcal{E}} - \frac{F_2 F_1^2}{2\mathcal{E}^2} - \frac{F_1^4}{24\mathcal{E}^3} \ ,$$

$$F_5 = \mathcal{E} Z_5 - \frac{F_4 F_1}{\mathcal{E}} - \frac{F_3 F_2}{\mathcal{E}} - \frac{F_3 F_1^2}{2\mathcal{E}^2} - \frac{F_2^2 F_1}{2\mathcal{E}^2} - \frac{F_2 F_1^3}{6\mathcal{E}^3} - \frac{F_1^5}{120\mathcal{E}^4} \ ,$$

$$\ldots\ldots$$

# Explicit results

- Expanding in instanton numbers, $F^{(\text{n.p.})} = \sum_k q^k F_k$, we have

$$
\begin{aligned}
F_1 &= 8\mathrm{P}f(\Phi)\ , \\
F_2 &= \mathcal{E} Z_2 - \frac{F_1^2}{2\mathcal{E}}\ , \\
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F_5 &= \mathcal{E} Z_5 - \frac{F_4 F_1}{\mathcal{E}} - \frac{F_3 F_2}{\mathcal{E}} - \frac{F_3 F_1^2}{2\mathcal{E}^2} - \frac{F_2^2 F_1}{2\mathcal{E}^2} - \frac{F_2 F_1^3}{6\mathcal{E}^3} - \frac{F_1^5}{120\mathcal{E}^4}\ , \\
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$$F_1 = 8\mathrm{P}f(\Phi)\ ,$$

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$$F_3 = \frac{32}{3}\,\mathrm{P}f(\Phi)\ ,$$

$$F_4 = \mathcal{E}Z_4 - \frac{F_3F_1}{\mathcal{E}} - \frac{F_2^2}{2\mathcal{E}} - \frac{F_2F_1^2}{2\mathcal{E}^2} - \frac{F_1^4}{24\mathcal{E}^3}\ ,$$

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$$F_3 = \frac{32}{3}\,\mathrm{P}f(\Phi)\,,$$

$$F_4 = \frac{1}{4}\mathrm{Tr}\Phi^4 - \frac{1}{4}\left(\mathrm{Tr}\Phi^2\right)^2\,,$$

$$F_5 = \frac{48}{5}\,\mathrm{P}f(\Phi)\,,$$

$$\ldots\ldots$$

# Explicit results

- The D-instanton induced effective “prepotential” is

$$F^{(\mathrm{n.p.})}(\Phi) = 8\,\mathrm{Pf}(\Phi)\left(q + \frac{4}{3}q^3 + \frac{6}{5}q^5 + \dots\right) + \mathrm{Tr}\Phi^4\left(\frac{1}{2}q^2 + \frac{1}{4}q^4 + \dots\right)$$
$$+ \left(\mathrm{Tr}\Phi^2\right)^2\left(\frac{1}{4}q^2 + \frac{1}{4}q^4 + \dots\right)$$

- It is natural to generalize these results and write

$$F^{(\mathrm{n.p.})}(\Phi) = 8\,\mathrm{Pf}(\Phi)\sum_{k=1} d_{2k-1}\,q^{2k-1} + \frac{1}{2}\mathrm{Tr}\Phi^4\sum_{k=1}\left(d_k\,q^{2k} - d_k\,q^{4k}\right)$$
$$+ \frac{1}{8}\left(\mathrm{Tr}\Phi^2\right)^2\sum_{k=1}\left(d_k\,q^{4k} - 2d_k\,q^{2k}\right)$$

with $\quad d_k = \sum_{\ell|k}\frac{1}{\ell}\quad$ sum over the inverse divisors of $k$

# Complete results

- Taking into account the contributions at tree-level for $\mathrm{Tr}F^4$ and at 1-loop for $\left(\mathrm{Tr}F^2\right)^2$, we can finally write the full expression for the quartic terms in the effective action of the D7-branes

$$2\, t_8\, \mathrm{Pf}(F)\, \log\left|\frac{\eta(\tau+1/2)}{\eta(\tau)}\right|^4 + \frac{t_8\, \mathrm{Tr}F^4}{4}\, \log\left|\frac{\eta(4\tau)}{\eta(2\tau)}\right|^4$$
$$+ \frac{t_8\, (\mathrm{Tr}F^2)^2}{16}\, \log\left(\mathrm{Im}\,\tau\, \mathrm{Im}\, U\, \frac{|\eta(2\tau)|^8\, |\eta(U)|^4}{|\eta(4\tau)|^4}\right)$$

with $q = e^{2\pi \mathrm{i}\tau}$

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with $q=e^{2\pi i\tau}$

- Written in this form, we recognize the same expression for the quartic terms in the $\mathrm{SO}(8)^4$ Heterotic String on $T_2$ if we replace

$$\tau:\text{axion-dilaton}\longleftrightarrow T:\text{Kähler structure of the 2-torus } T_2$$
$$\text{D-instantons}\longleftrightarrow\text{world-sheet instantons}$$

# Heterotic / Type I′ duality

- In the SO(8)$^4$ Heterotic String on $T_2$ the BPS-saturated quartic terms in the gauge field strength $F$ come from a 1-loop computation

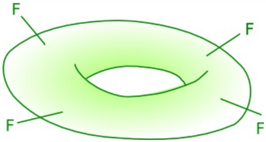
$$\frac{t_8\,\mathrm{Tr}F^4}{4}\,\log\left|\frac{\eta(4T)}{\eta(2T)}\right|^4$$

$$+\frac{t_8\,(\mathrm{Tr}F^2)^2}{16}\,\log\left(\mathrm{Im}\,T\,\mathrm{Im}\,U\,\frac{|\eta(2T)|^8\,|\eta(U)|^4}{|\eta(4T)|^4}\right)$$

Lerche+Stieberger, 1998; Gutperle, 1999; Kiritsis et al, 2000; ...

$$+2\,t_8\,\mathrm{Pf}(F)\,\log\left|\frac{\eta(T+1/2)}{\eta(T)}\right|^4$$

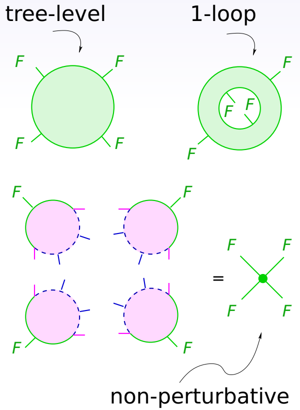
Gava et al, 1999



# Heterotic / Type I′ duality

- In the Type I′ theory the BPS-saturated quartic terms in $F$ have a tree-level (disk), 1-loop (annulus) and non-perturbative contributions ("integrated" mixed-disks) from D-instantons

$$\frac{t_8\,\mathrm{Tr}F^4}{4}\,\log\left|\frac{\eta(4\tau)}{\eta(2\tau)}\right|^4 + \frac{t_8\,(\mathrm{Tr}F^2)^2}{16}\,\log\left(\mathrm{Im}\,\tau\,\mathrm{Im}\,U\,\frac{|\eta(2\tau)|^8\,|\eta(U)|^4}{|\eta(4\tau)|^4}\right) + 2\,t_8\,\mathrm{Pf}(F)\,\log\left|\frac{\eta(\tau+1/2)}{\eta(\tau)}\right|^4$$



# Conclusions and perspectives

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- We have explicitly computed the non-perturbative effective couplings induced by stringy instantons in a simple setting, using the D7/D(–1) system in Type I′
- The method allows to compute also non-perturbative gravitational corrections: it is enough not to switch off the RR background $\mathcal{F}$ in the final expressions.
  - In this way one gets also the quartic terms $\mathrm{Tr}R^4$ and $\mathrm{Tr}R^2\mathrm{Tr}F^2$ in the effective action
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- These results perfectly match those of the $SO(8)^4$ Heterotic string and thus provide a non-trivial and explicit test of the Type I′/Heterotic duality
- The string instanton calculus is on solid ground also for the "exotic" configurations which have a very different spectrum of moduli as compared to ordinary gauge instantons

# Conclusions and perspectives

- Generalizations to "non-conformal" $d = 8$ models with gauge group $SO(2n)$ have been considered

  Fucito+Morales+Poghossian

- Is all this interesting and relevant for models in $d = 4$?
  - Explicit analysis of the stringy instanton effects in Type I′ orbifold compactifications with D7/D3/D(–1)-brane systems

    Billò+Frau+Fucito+A.L.+Morales+Poghossian, in progress

- Applications to $\mathcal{N} = 1$ in $d = 4$ models

- ...

# Conclusions and perspectives

Thank you!