A NEW CLASS OF $\mathcal{N}=2$ TOPOLOGICAL AMPLITUDES

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OUTLINE

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INTRODUCTION

CLASSIC EXAMPLE in $\mathcal{N}=2$ (Type II on CY)

$$F_g \sim \int_{\mathcal{M}_g} \left| (\mu G_-)^{3g-3} \right|^2$$

(BCOV)

$\mathcal{M}_g$ moduli space of genus g surfaces

μ BELTRAMI Differentials

G_- Supercurrent in the twisted $\mathcal{N}=2$ SCFT describing CY.

In the 4-Dim. Effective action, F_g give the couplings (AGNT, BCOV)

$$S \sim \int F_g W^{2g} \simeq \int F_g R^2 (T^2)^{g-1} + \cdots$$

W ~ Weyl superfield.

F_g depend on chiral vector ~~for~~ multiplets

F_g satisfy holomorphicity eq.
upto BOUNDARY TERMS IN $\mathcal{M}_g$
$\longrightarrow$ Holomorphic ANOMALY

APPLICATIONS

1) CORRECTIONS To the AREA LAW OF SUSY BLACK HOLE ENTROPY (OSV)

2) TESTS OF STRING DUALITIES

3) OPEN STRING - CLOSE STRING DUALITY NEAR THE CONIFOLD (OV)

(OPEN STRING) D-BRANES $\longleftrightarrow$ FLUX (CLOSED STRING)

GENERALIZATIONS

1) $\mathcal{N}=4$

TYPE II ON K3 (BERKOVITS, VAFA)
(OOGURI, VAFA)

WORLD SHEET $N=4$ SCFT

$$\begin{pmatrix} \tilde{G}^{+} \\ G^{+} \end{pmatrix} \underset{J^{++}}{\overset{J^{--}}{\rightleftarrows}} \begin{pmatrix} G^{-} \\ \tilde{G}^{-} \end{pmatrix}$$

$SU(2)$ currents
J^{++}, J, J^{--}

$$F_g \sim \int (\mu G^-)^{3g-3} \left(\int \tilde{G}^+\right)^{g-1} \int J$$

$\times$ Right movers

Note that in this case $U(1)$ anomaly in the twisted theory is $2g-2$

Hence the need to introduce $(\tilde{G}^+)^{g-1}$

In 6-Dim effective theory

$$F_g \, (R)^4 \, (T)^{4g-4}$$

There is an S^2 worth of choice for G^- and $\tilde{G}^+$ above

Obtained by means of Harmonic variables u_1, u_2 satisfying $|u_1|^2 + |u_2|^2 = 1$

$$\hat{G}^-_{(u)} = u_1 G^- + u_2 \tilde{G}^- \quad \text{etc.}$$

F_g satisfy <u>Harmonicity equation</u>

2) $\mathcal{N} = 1$

Heterotic or Type I on CY

$\downarrow$

Semitopological (twisting only in the supersymmetric sector) (AGNT)

$$F_g \sim \int (\mu G_-)^{3g-3} (\det Q)^2$$

$$S \sim \int F_g W^{2g} \qquad W = \text{chiral gauge superfield.}$$

$$S = \int F_g \, F^2 (\lambda^2)^{g-1} + \dots$$

$F \to$ Self dual gauge field strength

$\lambda \to$ gauginos (chiral)

HOLOMORPHICITY Eq does NOT close among F_g's

$\longrightarrow \quad F_{g,n}$

$$S \sim \int F_{g,n} \, W^{2g} \, \Pi^n$$

Π chiral projector on real functions of chiral fields

3) More recently

4 Dim. $N=4$ theory (Type II on $K_3 \times T^2$

or Heterotic on T^6)

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$$F_g \sim \int (\mu G_-(T^2))^g (\mu G_-(K_3))^{2g-4} (\mu J^{--}) \Psi_3$$

Total charge K3 sector

$$-(2g-4+2) = -(2g-2)$$

IN T^2 sector

$$(-g+1).$$

$$S = \int F_g (K^2)^g \sim \int F_g R^2 (T^2)^{g-1} (\partial\partial\varphi)^2$$

K is a superdescendant of Weyl superfield

Satisfy Harmonicity eq. w.r.t. SU(4) Harmonics (upto "anomalies")

4) New class of $\mathcal{N}=2$ topological terms

→ Type I on $K3 \times T_2$ (AHNS)

Can be obtained from Type II on $K3 \times T^2$ by considering involutions of genus g surfaces → world sheets with boundaries

→ Heterotic string on $K3 \times T^2$ (semi topological)

$$F_g \sim \int_{\mathcal{M}_g} (\mu G_-(T^2))^g (\mu G_-(K3))^{2g-4} (\mu J_{--}) \psi_3 \times (\det Q)^2$$

They give the couplings

$$F_g \; F^2 \; (\partial\lambda)^{g-2} \; (\partial\phi)^2$$

Self dual gauge Field Strength

Chiral gauginos

Chiral vector multiplet Scalar

F_g depend both on vector <u>and</u> Hyper moduli

Upto "anomalies" (Boundary terms) they satisfy

→ Holomorphicity with respect to vector moduli

→ Harmonicity eq. w.r.t. Hyper moduli

→ Second order Differential eq. w.r.t. Hyper moduli

EFFECTIVE ACTION

Harmonic variables $SU(2)/U(1)$

$$\begin{pmatrix} u_1^+ & u_1^- \\ u_2^+ & u_2^- \end{pmatrix} \in SU(2)$$

u_i^+, u_i^- have $U(1)$ charge ± 1

$$\bar{u}^i_+ \equiv \overline{(u_i^+)} \qquad \bar{u}^i_- \equiv \overline{(u_i^-)}$$

Harmonic functions are expansions in Harmonic variables u with definite $U(1)$ charge

Thus a charge $+1$ function

$$f^+(u) = f^i u_i^+ + f^{ijk} u_i^+ u_j^+ u_k^- + \ldots$$

u can be used to construct $1/2$ BPS multiplets that depend only on half the Grassmann coordinates

$$\theta^+_\alpha = \theta^i_\alpha u^+_i \qquad \bar{\theta}^{\dot\alpha}_- = \bar{u}^i_- \bar{\theta}^{\dot\alpha}_i$$

→ On-Shell Hyper multiplet

$$q^{+}(x^{\mu}, \theta^{+}, \bar{\theta}_{-}, u) = f^{i} u_{i}^{+} + \theta_{\alpha}^{+} \chi^{\alpha} + \bar{\psi}_{\dot{\alpha}} \bar{\theta}_{-}^{\dot{\alpha}} + \dots$$

f^{i} are scalars

χ & $\bar{\psi}$ are fermions.

→ From ~~onshell~~ vector multiplet

$$W = \underbrace{\varphi}_{\text{Complex Scalar}} + \underbrace{\theta_{\alpha}^{i} \lambda_{i}^{\alpha}}_{\text{gauginos}} + \theta_{\alpha}^{i} \theta_{\beta}^{j} \big(\underbrace{\epsilon_{ij} F_{+}^{\alpha\beta}}_{\text{Self dual Field Strength}} + \underbrace{\epsilon^{\alpha\beta} S_{(ij)}}_{\text{auxiliary}} \big)$$

Assuming On-shell $S_{(i,j)}$ vanish

$$\epsilon_{\alpha\beta} D_{i}^{\alpha} D_{j}^{\beta} W = 0 \qquad \text{(on-shell)}$$

we can construct a superfield

$$K_{-}^{\alpha} = D_{-}^{\alpha} W$$

K_{-}^{α} satisfies

$$D_{-}^{\beta} K_{-}^{\alpha} = \bar{D}_{\beta}^{+} K_{-}^{\alpha} = 0$$

$$\Rightarrow K_{-}^{\alpha}(\theta^{+}, \bar{\theta}_{-}, u) = \lambda_{i}^{\alpha} \bar{u}_{-}^{i} + i(\sigma^{\mu})^{\alpha\dot{\alpha}} \bar{\theta}_{\dot{\alpha}-} \partial_{\mu} \varphi + \theta_{\beta}^{+} F_{+}^{\alpha\beta} + \dots$$

Higher Derivative Coupling

$$S = \int d^4x \int du \int d^2\theta^+ d^2\bar{\theta}_- \, (K_- \cdot K_-)^{g-1} D_- D_- \tilde{F}_g$$

when $\tilde{F}_g(W, q^+, u)$

Assuming auxiliary fields vanish

$$(D_- \cdot D_-) \tilde{F}_g = (K_- \cdot K_-) F_g$$

This gives rise to

$$\int d^4x \int du \; F_+^2 (\partial\varphi)^2 (\lambda_- \cdot \lambda_-)^{g-2} \quad F_g(\varphi, f^+, u)$$

Now $(\lambda_- \cdot \lambda_-)^{g-2} = (\lambda_{i_1} \lambda_{i_2}) \cdots (\lambda_{i_{2g-3}} \lambda_{i_{2g-4}}) \, \bar{u}_-^{i_1} \cdots \bar{u}_-^{i_{2g-4}}$

SU(2) isospin $(g-2)$ and U(1) charge $= +(2g-4)$

Now F_g will have an expansion

$$F_g = \sum_n \xi(\varphi)_{i_1 \cdots i_{m+n}} \, \bar{u}_+^{i_1} \cdots \bar{u}_+^{i_{m+n}} \, (f^+)^n \qquad m = 2g-4$$

Total U(1) - Charge $= -m = -(2g-4)$

$$F_g = \sum_n \xi(\varphi)_{(i_1 \cdots i_{m+n})} \underbrace{\bar{u}_+^{i_1} \cdots \bar{u}_+^{i_{m+n}}}_{\substack{\downarrow \\ SU(2) \text{ isospin} \\ \left(\frac{m+n}{2}\right)}} \left(u^+_{j_1} \cdots u^+_{j_n}\right)\left(f^{j_1} \cdots f^{j_n}\right)$$

To get isospin $\left(\frac{m}{2}\right)$ to get non-vanishing integral over $\int du$

we need to contract n u^+_j's with n $\bar{u}_+^i$'s via δ^i_j to get singlets

So relevant part of F_g that survives $\int du$ integral is the <u>reduced function</u>

$$\tilde{F}_g = \sum_n \xi(\varphi)_{(i_1 \cdots i_{m+n})} \left(\bar{u}_+^{i_1} \cdots \bar{u}_+^{i_m}\right)\left(f^{i_{m+1}} f^{i_{m+2}} \cdots f^{i_{m+n}}\right)$$

Since $\xi^{(\phi)}_{(i_1 \cdots\cdots i_{m+n})}$ is totally Symmetric in $(i_1 \cdots i_{m+n})$ we have the following equations

1) $$\epsilon^{ij} \frac{\partial}{\partial \bar{u}^i_+} \frac{\partial}{\partial f^j_A} \mathcal{F}_g = 0$$

A denotes the species of hyper

<u>HARMONICITY Equation</u>

2) $$\epsilon^{ij} \frac{\partial}{\partial f^i_A} \frac{\partial}{\partial f^j_B} \mathcal{F}_g = 0$$

<u>SECOND ORDER Equation</u>

3) $$\frac{\partial \mathcal{F}_g}{\partial \bar{\varphi}_I} = 0$$

I denotes the vector multiplet

<u>Holomorphicity eq</u>

Coupling to Super gravity and using the fact that Hypermultiplet space is quaternionic one can show that

Harmonicity Eq.

$$\epsilon^{ij} \frac{\partial}{\partial \bar{u}^i_+} D_{jA} F_g = 0$$

↳ Covariant derivative

j is $SU(2)$ index

A is $Sp(n)$ index for some n

Second order equation

$SO(4;n)/SO(4) \times SO(n)$

$$\epsilon^{ij} D_{i,\hat{A}a} D_{j\hat{B}b} F_g = (g-1)\, \delta_{\hat{A},\hat{B}}\, \epsilon_{ab} F_g$$

$Sp(n)$ index A = $SO(n)$ index $\hat{A}$ and $SU(2)$ index a

$A \to (\hat{A}, a)$

II STRING COMPUTATION

$$\langle F_+^2 (\partial\varphi)^2 (\lambda^2)^{g-2} \rangle \sim F_g$$

in the RNS FORMULATION is

UNFORTUNATELY DIFFICULT

(Perhaps can be done in pure spinor formulation ?)

We compute another amplitude which is related to the above by supersymmetry

$$(K_- K_-)^g \xrightarrow{\text{Lowest Component}} (\lambda \cdot \lambda)^g$$

Saturate 2 θ^+ and 2 $\bar{\theta}_-$ by taking

2 χ_A's and 2 $\bar{\psi}_B$'s from Hypers

$$q_A^+ = f_A^+ + \theta_\alpha^+ \chi_A^\alpha + \bar{\psi}_{\dot{\alpha}A} \bar{\theta}_-^{\dot{\alpha}} + \dots$$

This will give 4-hyper derivatives of F_g

$$D^+_{A_1} D^+_{A_2} D^+_{A_3} D^+_{A_4} F_g .$$

# of Fermions $\quad 2g + 4$

$2g \rightarrow$ gauginos, $\quad 4 \rightarrow$ Hyperinos

$(-\frac{1}{2})$ ghost-picture $\longrightarrow -(g+2)$

On genus g Total ghost charge $= 2g-2$

$\Rightarrow$ # of Picture changing operators

$$= 2g-2 + (g+2) = 3g$$

Vertices	numbers	φ_1	φ_2	φ_3	H
gaugino	g	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{\sqrt{2}}$
gaugino	g	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{\sqrt{2}}$
χ_{A_1}	1	$-+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0
χ_{A_2}	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0
$\overline{\psi}_{A_3}$	1	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0
$\overline{\psi}_{A_4}$	1	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0
PCO	g	0	0	-1	0
PCO	$2g$	0	0	0	$-\frac{1}{\sqrt{2}}$

ϕ_1, ϕ_2 are bosonization of space-time fermions

ϕ_3 is bosonization of T^2 fermion

H is the bosonization of $U(1) \subset SU(2)$ current algebra of $N=4$ SCFT describing K3

Can Perform Spin-Structure Sum

The result is

$$D_{+A_1} D_{+A_2} D_{+A_3} D_{+A_4} \mathcal{F}_g$$

where

$$\mathcal{F}_g = \int_{\mathcal{M}_g} \Big\langle \big(\mu G_-(T_2)\big)^g \big(\mu\, G_-(K3)\big)^{2g-4} \big(\mu J_{--}\big)_{N_3} \; (\det Q)^2 \Big\rangle$$

in Heterotic theory

in the twisted internal theory

At a generic pt. in the moduli space
We have checked the Harmonicity,
Holomorphicity equations using
$\mathcal{N}=4$ super conformal algebra

HARMONICITY Eq

$$e^{ij}\frac{\partial}{\partial \bar{u}^i_+} D_{JA} F_g = \sum_{g_1=2}^{g-2} D_{A_+} D_{B_+} F_{g_1} \Omega^{BC} D_{C+} F_{g-g_1}$$

$$+ F_{1,AB}\, \Omega^{BC} D_{C+} F_{g-1}$$

$$+ F^{g-1}_{A\bar{K}}\, G^{\bar{K}L} D_L h^{(1)}$$

→ $h^{(1)}$ is the 1-loop threshold correction to gauge-couplings

→ $F^{g-1}_{A,\bar{K}}$ is a non-holomorphic coupling in the effective action which contributes to this physical amplitude via elimination of auxilary fields

$$S_{(ij)I} \sim \partial_I h_{JK}\, \chi^{(i}_J \chi^{j)}_K + \cdots$$

Holomorphicity eq.

$$\frac{\partial F_g}{\partial \bar{\varphi}^I} = \mathcal{F}^{g-1}_{\bar{I},\bar{K}} \; G^{\bar{K}L} \, \partial_L h^{(1)}$$

$\downarrow$

non-holomorphic coupling

Second-Order eq.

To avoid handling dangerous contact terms we calculated it in an Orbifold model with Hypermultiplet in $SO(4,n)/SO(4)\times SO(n)$

$K_3 \sim T^4/Z_2$ where Z_2 comes with a shift in T^2 so that twisted sectors become massive

$$\epsilon^{ij} D_{i(\hat{A},a)} D_{j(\hat{B},b)} F_g = (g-1)\, \delta_{\hat{A}\hat{B}}\, \epsilon_{ab} \int F_g$$

$$+ \text{ Boundary terms}$$

CONCLUSIONS

→ A NEW CLASS OF $N=2$ TOPOLOGICAL COUPLING which depends on Both Vector & Hyper moduli

→ They satisfy holomorphicity w.r.t. vector moduli and harmonicity (+ Second order equation) w.r.t. hyper moduli

Upto BOUNDARY TERMS

When gauge kinetic function is non-trivial

These BOUNDARY TERMS Give Rise to another sequence of non-holomorphic couplings.

OPEN QUESTIONS

→ What do these amplitudes compute mathematically?

→ what are there physical applications

Black hole entropy?

Partial or Complete Breaking of SUSY