F-theory Compactifications for SUSY GUTs

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based on work with Joe Marsano and Natalia Saulina

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Plan

1. Features of Local F-theory GUTs
2. Refined Local Models
   - Monodromies and Spectral Cover
   - Factorized Spectral Cover and Constraints
3. A Compact Geometry
4. Outlook

1. Features of Local F-theory GUTs [cf. talks by Vafa and Heckman]

F-theory GUTs [Donagi, Wijnholt] [Beasley, Heckman, Vafa]:
F-theory on $\mathbb{R}^{1,3} \times X_4$, X_4 = elliptically fibered CY4 over B_3:

$$\begin{array}{ccc} \mathbb{E}_\tau & \rightarrow & X_4 \\ & & \downarrow \\ & & B_3 \supset S_{\rm GUT} \end{array}$$

Locally: ALE fibration over 4-cycle $S_{\rm GUT}$.

- Local $SU(5)$ singularity: GUT gauge degrees of freedom
- Matter and Higgs fields: rank 1 enhancements along curves

$$\begin{array}{rl} SU(6): & \overline{\mathbf{5}}_M \,, \; \mathbf{5}_H \,, \; \overline{\mathbf{5}}_H \\ SO(10): & \mathbf{10}_M \end{array}$$

- Yukawas: rank 2 enhancements $SO(12)$ and E_6 enhancement

$$W \supset \lambda_{\rm bottom}\, \overline{\mathbf{5}}_H \times \overline{\mathbf{5}}_M \times \mathbf{10}_M + \lambda_{\rm top}\, \mathbf{5}_H \times \mathbf{10}_M \times \mathbf{10}_M$$

- Flavor structure: most natural to have all generations arise from single matter curves [Heckman, Vafa]

$$\Sigma_{\bar{\mathbf{5}}_M} : \quad 3 \times \bar{\mathbf{5}}_M \,, \qquad \Sigma_{\mathbf{10}_M} : \quad 3 \times \mathbf{10}_M$$

- GUT-breaking by hypercharge flux F_Y requires

$$F_Y|_{\Sigma_M} = 0 \,, \qquad F_Y|_{\Sigma_{\mathbf{5}_H}} = +1 \,, \qquad F_Y|_{\Sigma_{\bar{\mathbf{5}}_H}} = -1$$

Masslessness of $U(1)_Y$: [Buican, Malyshev, Morrison, Verlinde, Wijnholt]

$\Rightarrow$ F_Y is dual in S_{GUT} to 2-cycle, that is homologically trivial in B_3

- Absence of dimension 4 proton decay operators:

$$\mathbf{10}_M \times \bar{\mathbf{5}}_M \times \bar{\mathbf{5}}_M \qquad \text{and} \qquad \mathbf{10}_M \times \bar{\mathbf{5}}_H \times \bar{\mathbf{5}}_H$$

- Absence of tree-level μ-term $\mu H\bar{H}$: $U(1)_{PQ}$

[Marsano, Saulina, SS-N][Heckman, Vafa]

What controls the superpotential?

In local models:
superpotential dictated by independent $U(1)$ gauge symmetries

In a global setup:
identifications of $U(1)$s by monodromies

$\Rightarrow$ Spectral Cover keeps track of all symmetries underlying the theory

2. Refined Local Models [Donagi, Wijnholt]

<ins>Favorable Flavor:</ins> E_8 gauge symmetry

$$E_8 \rightarrow SU(5)_\perp \times SU(5)_{\text{GUT}}$$

$$\mathbf{248} \rightarrow (\mathbf{24},\mathbf{1}) + (\mathbf{1},\mathbf{24}) + (\overline{\mathbf{10}},\mathbf{5}) + (\overline{\mathbf{5}},\overline{\mathbf{10}}) + (\mathbf{10},\overline{\mathbf{5}}) + (\mathbf{5},\mathbf{10})$$

<ins>$SU(5)_\perp$ weights:</ins> $\mathbf{5}$: $\lambda_i = \sum_{k=1}^{i} \alpha_{5-k}$ with $\sum_{i=1}^{5} \lambda_i = 0$,

$\mathbf{10}$: $\lambda_i + \lambda_j$

- <ins>Matter/Higgses:</ins>

  $\mathbf{10}$: $SO(10)$: λ_i

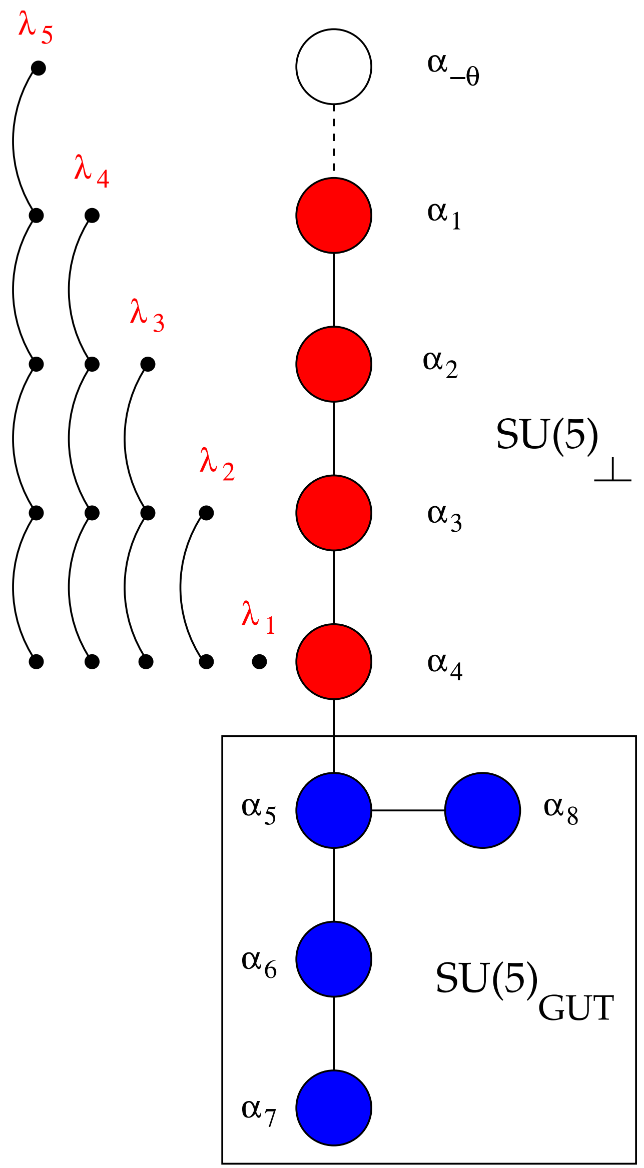
  $\overline{\mathbf{5}}$: $SU(6)$: $\lambda_i + \lambda_j$

- <ins>Yukawas:</ins>

  Bottom: $SO(12)$: $(\lambda_i + \lambda_j) \cdot (\lambda_k + \lambda_l) \cdot (\lambda_m)$

  Top: E_6: $(\lambda_i) \cdot (\lambda_j) \cdot (-\lambda_i - \lambda_j)$

- Flavor (CKM, PMNS): E_8 [Heckman, Tavanfar, Vafa]



Geometric implementation

Our starting point:

Local Geometry over S_{GUT} is deformed E_8 singularity

$$y^2 = x^3 + b_5 xy + b_4 x^2 z + b_3 yz^2 + b_2 xz^3 + b_0 z^5$$

- λ_i: encode volumes of blow-up $\mathbb{P}^1$'s
- b_n depend on canonical and normal bundle of S_{GUT} and are related to volumes of $\mathbb{P}^1$'s by

$$b_n(\lambda_i) = e_n(\lambda_i) b_0$$

$e_n =$ elementary symmetric polys

Geometric implementation

Deformed E_8 singularity

$$y^2 = x^3 + b_5 xy + b_4 x^2 z + b_3 y z^2 + b_2 x z^3 + b_0 z^5$$

Various enhancement loci translate into: [Andreas, Curio]

$SU(5)$:		Resolve $\mathbb{P}^1$ associated to $\alpha_{5,6,7,8}$
$SO(10)$:	**10** matter	$0 = b_5 \sim \prod_i \lambda_i$
$SU(6)$:	$\overline{\mathbf{5}}$ matter	$0 = P = b_0 b_5^2 - b_2 b_3 b_5 + b_3^2 b_4 \sim \prod(\lambda_i + \lambda_j)$
$SO(12)$:	Bottom:	$0 = b_5 = b_3$
E_6 :	Top:	$0 = b_5 = b_4$
E_8 :		$0 = b_2 = b_3 = b_4 = b_5$

Monodromies

$E_8 \to SU(5) \times U(1)^4$: naively expect four $U(1)$ gauge symmetries.

However geometry specified by $b_n = b_0 e_n(\lambda_i)$

$\Rightarrow$ Inversion $\lambda_i(b_n)$ generically has branch-cuts

$\Rightarrow$ monodromy group $G \subset S_5$= Weyl group of $SU(5)_\perp$ acts on λ_i

$\Rightarrow$ Spectral cover encodes monodromies

$$\mathcal{C}_{\mathbf{10}} : \qquad b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 \ \sim \ b_0 \prod_{i=1}^{5} (\lambda_i + s) = 0$$

[Hayashi, Kawano, Tatar, Watari] [Donagi, Wijnholt]

Projectivized version:

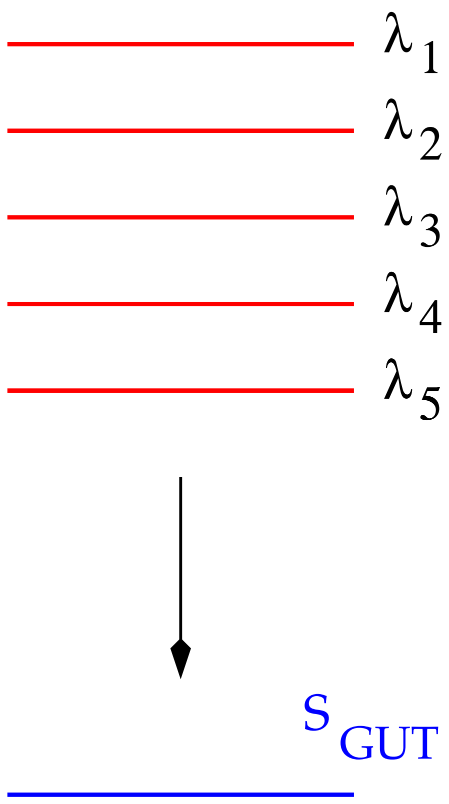
$$\mathcal{C}_{\mathbf{10}} \ : \ b_0 U^5 + b_2 V^2 U^3 + b_3 V^3 U^2 + b_4 V^4 U + b_5 V^5 = 0$$

Spectral Cover and $U(1)$ gauge symmetries

- Spectral cover is an auxiliary space, encodes all monodromies.
- $\mathcal{C}_{\mathbf{10}}$ is 5-fold cover of S_{GUT}:

$$b_0 U^5 + b_2 V^2 U^3 + b_3 V^3 U^2 + b_4 V^4 U + b_5 V^5 = 0$$

- Monodromy group $G \subset S_5$ acts on sheets and identifies $U(1)$'s.



Matter Curves in $\mathcal{C}_{\mathbf{10}}$

Matter curves naturally live inside $\mathcal{C}_{\mathbf{10}}$:

$$b_0 U^5 + b_2 V^2 U^3 + b_3 V^3 U^2 + b_4 V^4 U + b_5 V^5 = 0$$

10 matter curve: $b_5 = 0 \Leftrightarrow \mathcal{C}_{\mathbf{10}} \cap (U = 0)$

$\bar{\mathbf{5}}$ matter curve:
- Usually: $\bar{\mathbf{5}}$ matter curve in anti-symmetric spectral cover $\mathcal{C}_{\mathbf{5}}$
- Want to keep track of charges of $\bar{\mathbf{5}}$ fields
- Embed into $\mathcal{C}_{\mathbf{10}}$:

  Auto $\tau : \lambda_i \to -\lambda_i$ or $V \to -V$. Fixed locus:

$$\begin{aligned}
\mathcal{C}_{\mathbf{10}} \cap \tau \mathcal{C}_{\mathbf{10}} &= (\lambda_i = 0) \cup (\lambda_i + \lambda_j = 0) \cup (\lambda_i \to \infty) \\
&= \Sigma_{\mathbf{10}} \cup \Sigma_{\bar{\mathbf{5}}} \cup \Sigma_\infty \\
&= \mathcal{C}_{\mathbf{10}} \cap (U) \cup \mathcal{C}_{\mathbf{10}} \cap (\mathcal{C}_{\mathbf{10}} - (U) - (3V)) \cup \mathcal{C}_{\mathbf{10}} \cap (3V)
\end{aligned}$$

Factorization of Spectral Cover

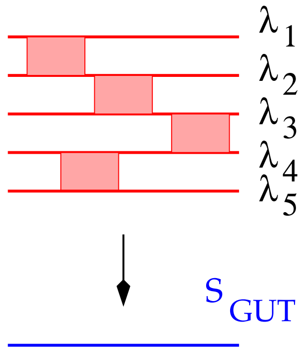
[Tatar, Tsuchiya, Watari], [Marsano, Saulina, SS-N]

First thing to read off from the spectral cover:

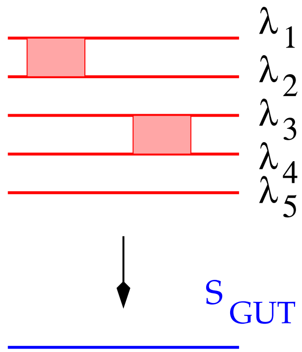
Independent gauged $U(1)$ symmetries are encoded in # factors of $\mathcal{C}_{\mathbf{10}}$

$U(1)$ gauge bosons are elements in Cartan subalgebra:

- $G =$ transitive subgroup of S_5:
  only invariant combination is $\sum_{i=1}^{5} \lambda_i = 0$
  $\Rightarrow$ no gauged $U(1)$



- λ_i in reducible representation of G:
  $\mathcal{C}_{\mathbf{10}}$ factors into N components $\Rightarrow (N-1)$
  gauged $U(1)$s



Refined structure from Spectral Cover

Example: $G = \mathbb{Z}_4 =< (1234) >$:

Decomposition of λ_i under $G = C_4$:

$$\{\lambda_i\} \rightarrow \mathbf{10}^{(1)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} + \mathbf{10}^{(2)}_{\lambda_5}$$

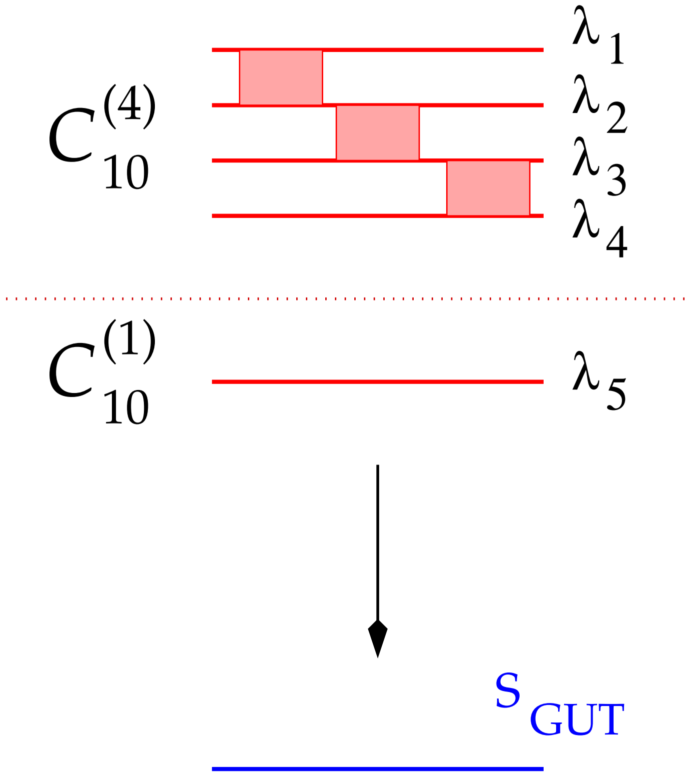
$$\{\lambda_i + \lambda_j\} \rightarrow \overline{\mathbf{5}}^{(1)}_{\lambda_1+\lambda_2,\cdots} + \overline{\mathbf{5}}^{(2)}_{\lambda_1+\lambda_3,\cdots} + \overline{\mathbf{5}}^{(3)}_{\lambda_1+\lambda_5,\cdots}$$

E_6 and $SO(12)$ points yield:

$$\mathbf{10}^{(1)} \times \mathbf{10}^{(1)} \times \mathbf{5}^{(1)}, \quad \mathbf{10}^{(1)} \times \mathbf{10}^{(1)} \times \mathbf{5}^{(2)}$$

$$\mathbf{10}^{(2)} \times \overline{\mathbf{5}}^{(1)} \times \overline{\mathbf{5}}^{(1)}, \quad \mathbf{10}^{(2)} \times \overline{\mathbf{5}}^{(2)} \times \overline{\mathbf{5}}^{(2)},$$

But: $\mathbf{10}^{(2)} \times \overline{\mathbf{5}}^{(1)} \times \overline{\mathbf{5}}^{(2)}$ forbidden.



We can assign $U(1)$ charges to get all the allowed couplings. But there is no assignment of charges that excludes $\mathbf{10}^{(2)} \times \overline{\mathbf{5}}^{(1)} \times \overline{\mathbf{5}}^{(2)}$. This requires knowledge of the monodromies.

Putting the spectral cover to use:

Phenomenological wish list:

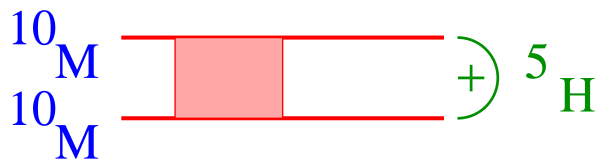
- no exotics, 3-generations
- top and bottom Yukawas
- flavor structure
- no tree-level μ-term...

$\Rightarrow$ What do these constraints imply for the spectral cover $\mathcal{C}_{\mathbf{10}}$?

Constraints on $\mathcal{C}_{\mathbf{10}}$

[Marsano, Saulina, SS-N]

1. <u>Top-Yukawa $\mathbf{10}_M \times \mathbf{10}_M \times \mathbf{5}_H$:</u>



$$(\lambda_i) = (\lambda_j) = (\lambda_i + \lambda_j) = 0$$

$\Rightarrow \mathcal{C}^{10_M}$ at least rank 2 and contains $\Sigma_{\mathbf{5_H}}$

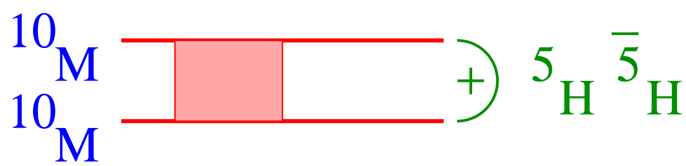
$\Rightarrow$ $\mathbf{10}_M$ and $\mathbf{5}_H$ in same component of $\mathcal{C}_{\mathbf{10}}$

⋮

2. <u>Hypercharge Flux Constraint:</u>

No exotics

$$\Rightarrow \qquad F_Y|_{\mathbf{10}^i} = 0\,, \quad \forall i$$



$\Rightarrow F_Y|_{\bar{\mathbf{5}}} = 0$ on net $\bar{\mathbf{5}}$ on each factor of $\mathcal{C}$

$\Rightarrow$ $\mathbf{10}_M$, $\mathbf{5}_H$, $\bar{\mathbf{5}}_H$ from single component $\mathcal{C}^{10_M}$

⋮

NB: Higgses arise from a single matter curve, which then has to factor further, in H_u and H_d.

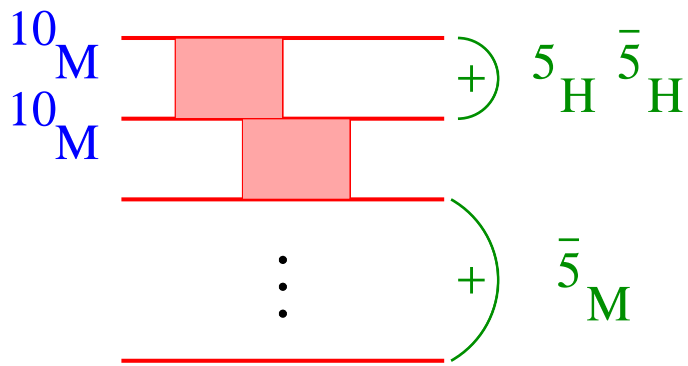
Constraints on $\mathcal{C}_{\mathbf{10}}$ (cont.)

3. Bottom-Yukawa $\overline{\mathbf{5}}_H \times \overline{\mathbf{5}}_M \times \mathbf{10}$:

$$(-\lambda_i - \lambda_i) = (-\lambda_k - \lambda_l) = (\lambda_m) = 0$$

$\Rightarrow$ From 2.) $\overline{\mathbf{5}}_H$ in same component of $\mathcal{C}_{\mathbf{10}}$

$\Rightarrow$ $\mathcal{C}^{10_M}$ has to have at least 3 sheets



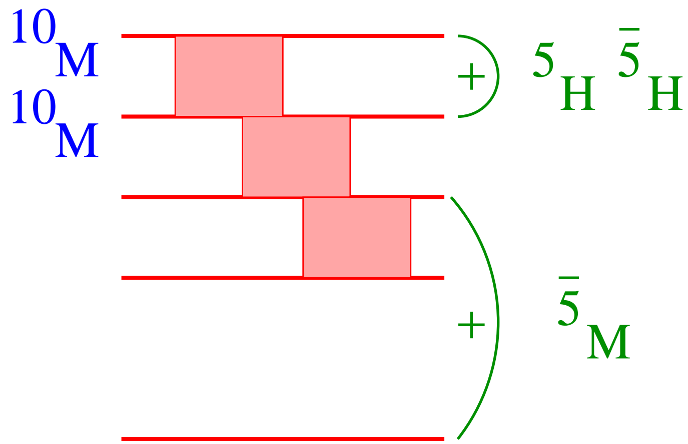
4. Absence of tree-level μ-term:

Exclude tree-level $\mu H_u H_d$

$\Rightarrow$ Only factorization realizing this is

$$\mathcal{C}_{\mathbf{10}} = \mathcal{C}_{\mathbf{10}}^{(4)} + \mathcal{C}_{\mathbf{10}}^{(1)}$$

Accidental global $U(1)_{PQ}$: $PQ(H_u) = PQ(H_d)$



Constraints on $\mathcal{C}_{\mathbf{10}}$ (cont.)

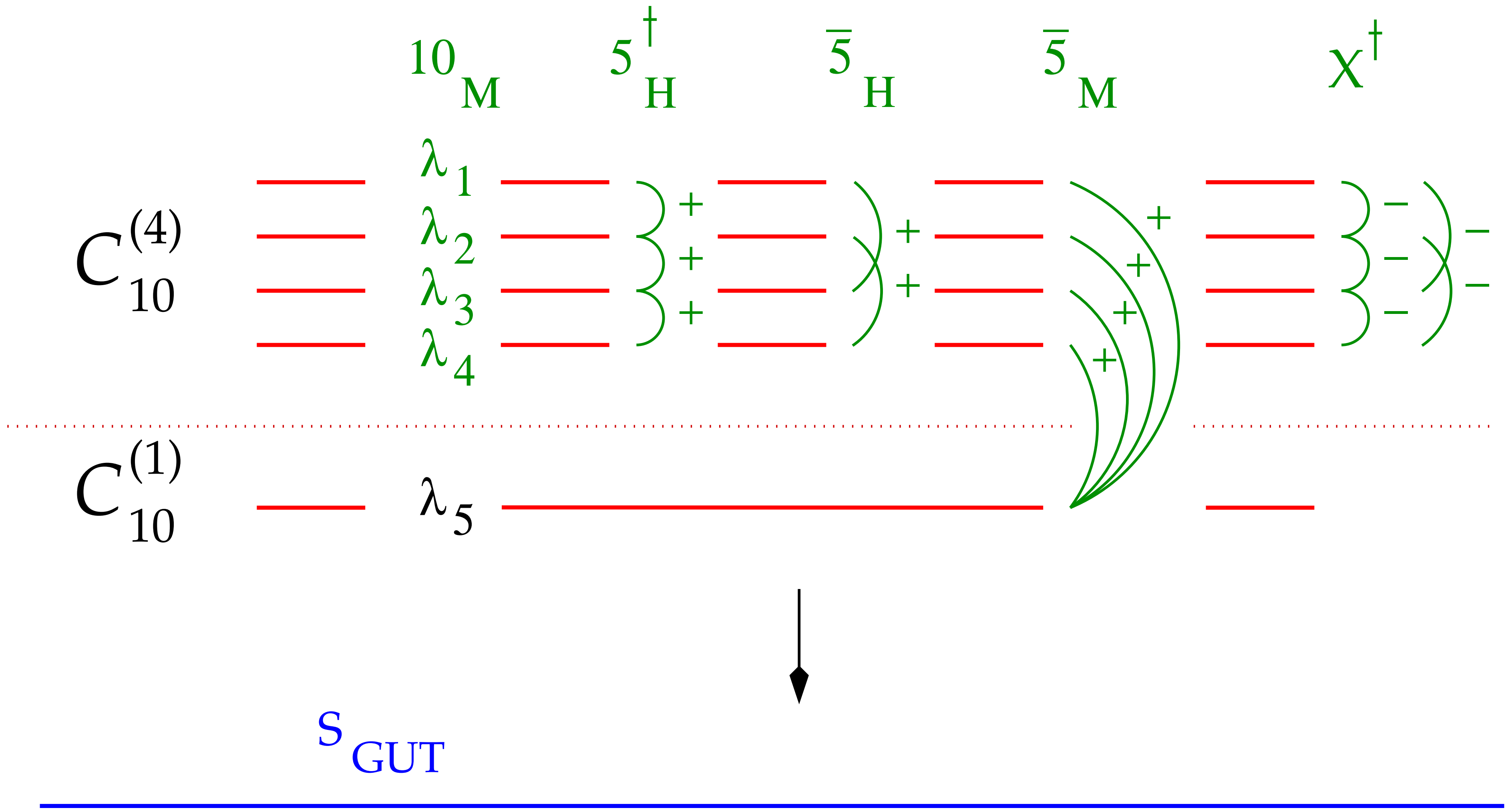
Summary: [Marsano, Saulina, SS-N]

Yukawas, no exotics and no tree-level $\mu \Rightarrow$ 4+1 factorization $\mathcal{C}_{\mathbf{10}}^{(4)} + \mathcal{C}_{\mathbf{10}}^{(1)}$

Comments:

- Absence of tree-level μ-term requires : $G = D_4$ or $\mathbb{Z}_4$ or Klein$_4$
- Only one $U(1)$ gauge symmetry, combination of $U(1)_Y$ and $U(1)_{B-L}$
- No gauged $U(1)_{PQ}$
- Problems with neutrinos
  $\Rightarrow$ Can we relax some constraint?
  $\Rightarrow$ Different mechanism for removing exotics?

4+1 Factorized Spectral Cover: $G = \mathbb{Z}_4$



Charges: **10**: $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, **5**$_H$: $(-\lambda_1 - \lambda_2), (-\lambda_2 - \lambda_3), (-\lambda_3 - \lambda_4)$, etc.

Properties of the $4+1$ factorized spectral cover

$$\mathcal{C}^{(4)} + \mathcal{C}^{(1)} : \qquad (a_0 U^4 + a_1 U^3 V + a_2 U^2 V^2 + a_3 U V^3 + a_4 V^4)(d_0 U + V) = 0$$

with $b_1 = a_0 + a_1 d_0 = 0$.

Matter curves:

$$\mathbf{10}: \quad a_4 = 0\,, \qquad \bar{\mathbf{5}}: \quad P = (a_3(a_2 + a_3 d_0) - a_1 a_4)(a_2 + d_0(a_3 + a_4 d_0)) = 0$$

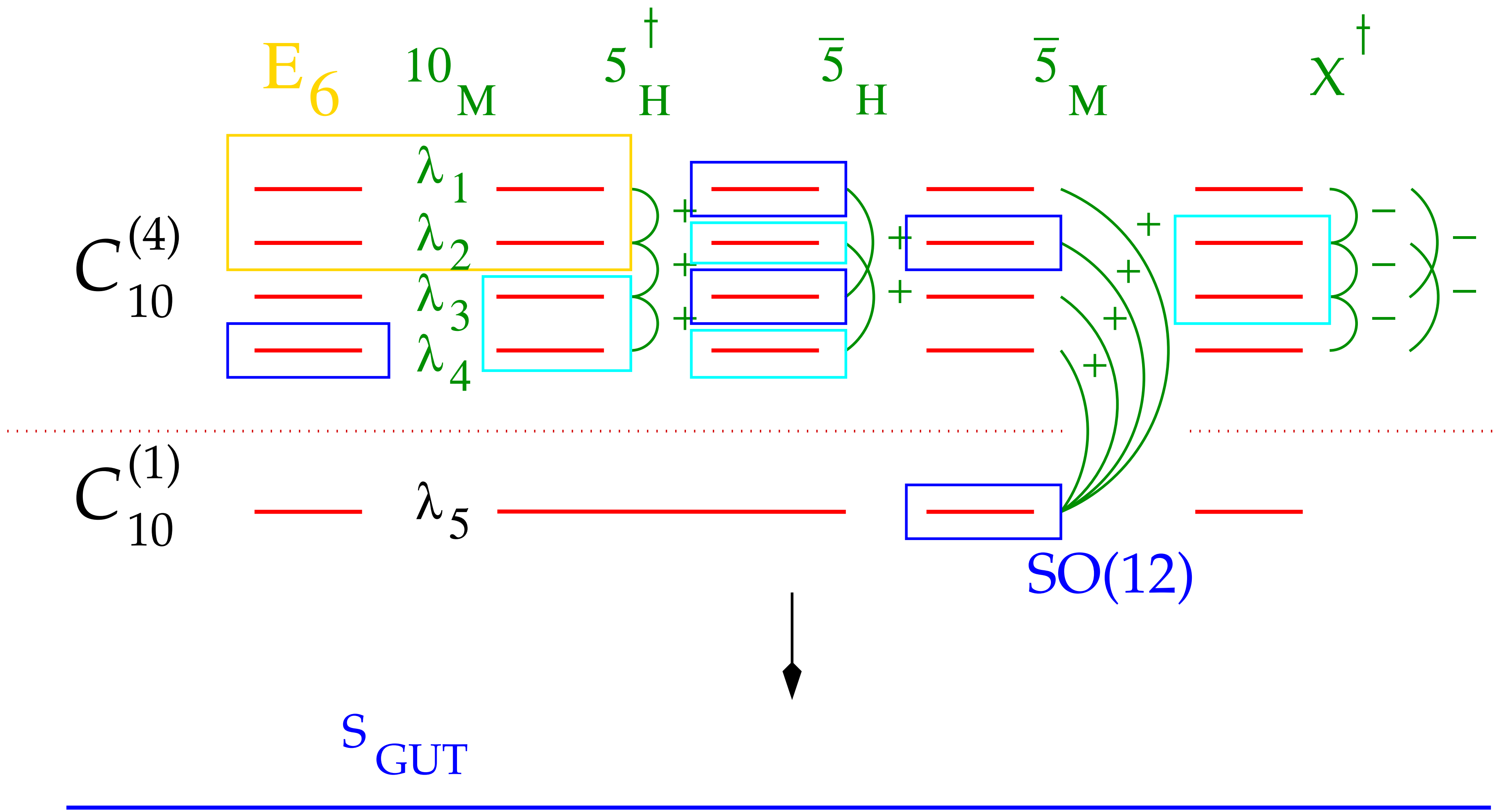
Note: Automatically factorized $\bar{\mathbf{5}}$ matter curve: $P = P_H P_M$.

Yukawas:

$$SO(12): \quad a_4 = a_2 + a_3 d_0 = 0\,, \qquad E_6: \quad a_4 = a_3 = 0$$

$\Rightarrow$ This factorization automatically guarantees correct Yukawa couplings

Yukawa couplings in 4+1 factorization



$$\int d^2\theta \left(\mathbf{10}_M \times \mathbf{10}_M \times \mathbf{5}_H + \bar{\mathbf{5}}_H \times \bar{\mathbf{5}}_M \times \mathbf{10}_M\right) + \int d^4\theta \frac{1}{M_{\rm GUT}} X^\dagger H \bar{H}$$

Summary so far

General semi-local analysis:

- Starting point: Deformed E_8 singularity over S_{GUT}: b_i
- Spectral cover: $\prod(s + \lambda_i) = 0$
- Monodromies act on spectral cover
- Independent $U(1)$ gauge symmetries encoded in # of factors of spectral cover
- Fundamental phenomenological requirements for $SU(5)$ GUTs $\Rightarrow$ 4+1 factorized spectral cover

3. A Compact Geometry

[Marsano, Saulina, SS-N]

Aim: Explicit realization of 4+1 model in compact CY four-fold
$\Rightarrow$ specify explicit sections b_n

Recall: X_4 = elliptically fibered CY4 with three-fold base B_3:

$$\begin{array}{ccc} \mathbb{E}_\tau & \longrightarrow & X_4 \\ & & \downarrow \\ & & B_3 \supset S_{\text{GUT}} \end{array}$$

Constraints on B_3:

- X_4 Calabi-Yau $\Rightarrow$ B_3 Fano i.e. $K_{B_3}^{-1}$ ample
- Hypercharge constraint: F_Y dual in S_{GUT} to class that is trivial in B_3
- S_{GUT} is a del Pezzo surface

Hypercharge flux constraint

[Buican, Malyshev, Morrison, Verlinde, Wijnholt]

[Beasley, Heckman, Vafa], [Donagi, Wijnholt]

Let $S = dP_n$ and

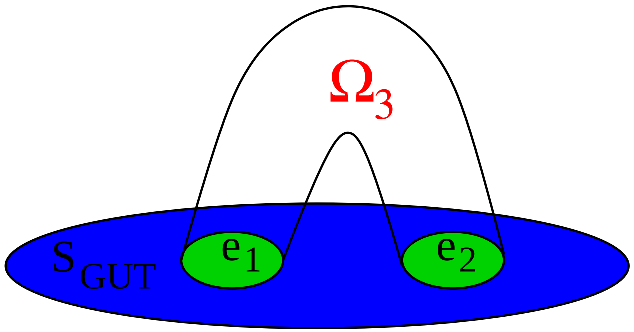
$$H_2(dP_n, \mathbb{Z}) = \langle h, e_1, \cdots, e_n \rangle \, , \qquad h^2 = 1 \, , \quad e_i \cdot e_j = -\delta_{ij}$$

Representative for hypercharge flux

$$[F_Y] = e_1 - e_2$$

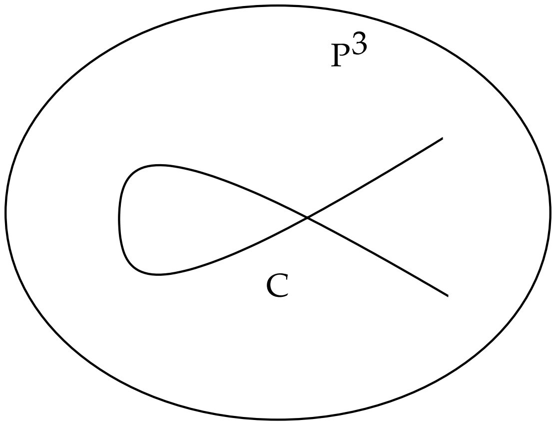
Hypercharge $U(1)_Y$ remains massless
if there is a 3-chain Ω_3 in B_3:

$$\partial\Omega_3 = e_1 \cup (-e_2)$$



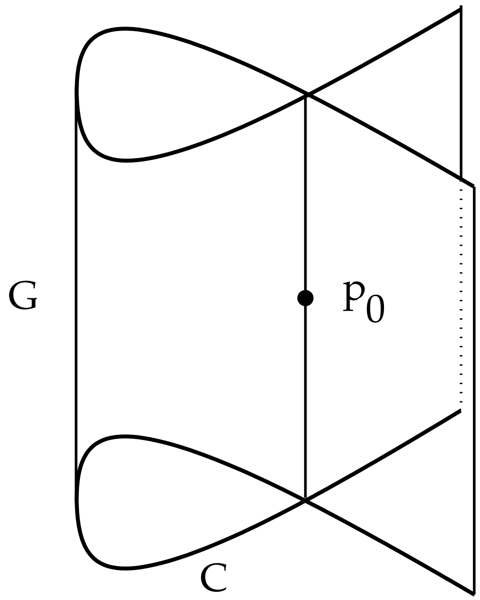
Three-Fold Construction

[Marsano, Saulina, SS-N]



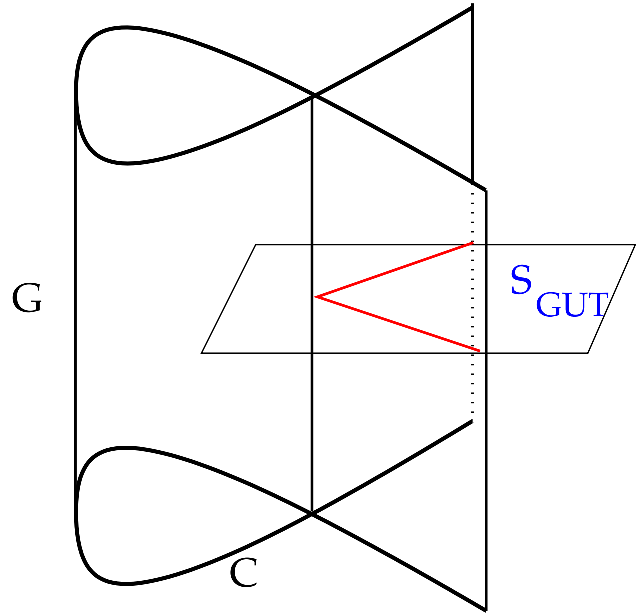
1. Starting point: nodal curve C, locally

$$xy = z = 0$$



2. Blowup along C: Conifold singularity

$$xy = zu$$



3. Blowup conifold to $S_{GUT} = \mathbb{P}^1 \times \mathbb{P}^1$

$\mathbb{P}^1$'s homologous in 3-fold

Local construction embeddable into $\mathbb{P}^3$. B_3 automatically Fano.

Exceptional divisor $S_{\text{GUT}} = \mathbb{P}^1_{(1)} \times \mathbb{P}^1_{(2)}$ has required properties of S_{GUT}:

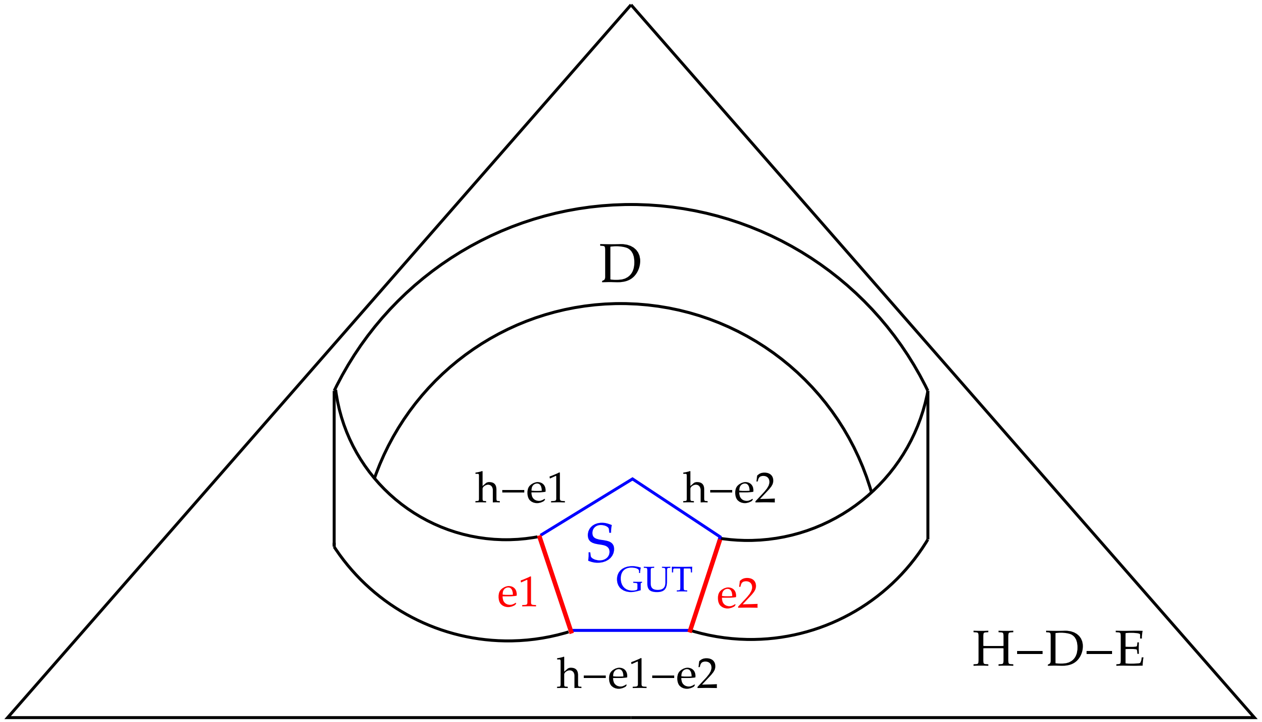
$$\mathbb{P}^1_{(1)} \sim \mathbb{P}^1_{(2)} \qquad \text{in} \qquad H_2(B_3, \mathbb{Z})$$

NB: Flopping the curve G yields dP_2 divisor with same property.

Divisors: $H = dP_3,\ D = \mathbb{F}_4\ , E = S_{\text{GUT}} = dP_2$

Final 3-fold: All holomorphic sections explicitly constructed

[Marsano, Saulina, SS-N]



Three-Generation Model

[Marsano, Saulina, SS-N]

Consider now 4+1 factorized model in this geometry.

Matter curves:

$$\mathbf{10}: \quad a_4 = 0\,, \qquad \overline{\mathbf{5}}: \quad (a_3(a_2 + a_3 d_0) - a_1 a_4)(a_2 + d_0(a_3 + a_4 d_0)) = P_H\, P_M = 0$$

Classes of a_i are fixed in terms of $K_{S_{\rm GUT}}$ and $N_{S_{\rm GUT}|B_3}$.
In our explicit example these are given by:

$$\begin{aligned} [\Sigma_{10}]|_{S_{\rm GUT}} &= 2h - (e_1 + e_2) \\ [\Sigma_{5,H} + \Sigma_{\bar{5},H}]|_{S_{\rm GUT}} &= 13h - 5(e_1 + e_2) \\ [\Sigma_{5,M}]|_{S_{\rm GUT}} &= 8h - 3(e_1 + e_2) \end{aligned}$$

G-fluxes naturally come from spectral cover [Donagi, Wijnholt]

Constructed fluxes in spectral cover s.t. $3 \times \mathbf{10}_M$ and $3 \times \overline{\mathbf{5}}_M$, no net chiral matter on Higgs curve. [Marsano, Saulina, SS-N]

$\Rightarrow$ 3-generation $SU(5)$ GUT in compact CY4.

4. Dones...

All following features are realized in our compact geometry:

- Exotic-free $SU(5)$ GUT
- 3 generations of **10** and of $\bar{\mathbf{5}}$ on single matter curve, each
- GUT-breaking by hypercharge flux F_Y
- $U(1)_Y$ is ensured to remain massless
- No dimension 4 proton decay

... and ToDos

- $G = S_4$ so far: require refinement to $G = D_4, \mathbb{Z}_4, \text{Klein}_4$
  $\Rightarrow$ global $U(1)_{PQ}$ yet
  $\Rightarrow$ μ-problem, dimension 5 proton decay
  $\Rightarrow$ technical obstruction: further refinement of spectral cover so that $G = D_4, \mathbb{Z}_4, \text{Klein}_4$
- Relax constraints on factorization?
  $\Rightarrow$ Requires different mechanism to lift exotics
  $\Rightarrow$ Could yield gauged $U(1)_{PQ}$
- SUSY breaking: further effects due to gravity mediation?
- Moduli stabilization

Thank you