2+1 Topics

1) Cosmology

2) Insightful D-branes
   W/ Horowitz, Lawrence

3) Executive Summary
   (papers online)

3) F is for Four
   W/ Polchinski

New (in progress)
String theory as Rocket Science

CMB observables
  - gravity waves (B mode polarization)
  - non-Gaussianity
are UV Sensitive

\[ \Delta \Phi \sim \left( \frac{r}{0.01} \right)^\frac{1}{4} \]

\[ r = \frac{\text{tensor}}{\text{scalar}} \]

Must control contributions to inflaton effective action at \( \geq \) dimension 6
  - D3-\overline{D3} Warping, tuning of \( V_{\chi}^n M_p^2 \)
  - Monodromy \( \exists \) symmetry \( \rightarrow r \sim 0 \)
  - trapping \( \rightarrow \) new source of \( \Phi \), non-Gaussian
  - DBI \( \rightarrow \) N.G. \cdot Nflation \cdot Kähler
- Refs: Review papers
- CMB Pol Inflation working group report '08
- Planck launch + 3 years = 2012
- KITP works on Primordial Cosmology E5, M. Zaldamiaga
Insightful D-branes

C. Horowitz
A. Lawrence

- Poincare Patch
- AdS/CFT duality

on compact hyperbolic spatial slices governs physics inside the horizon of a black hole.
We'd like a non-perturbative formulation of $4d$ physics:

\[
\text{\{ internal \}} \quad \leftrightarrow \quad 4d
\]

with a hierarchy of energy scales

\[
\frac{1}{M_{\text{KK}}^{\text{string}}}_{\text{0}} \gg \frac{1}{L_{4d}}
\]
Outline

. The problem & previous attempts
. Our strategy
. Consistency conditions (cf Singularities)
. Candidate Examples

$\text{AdS}_5 \times \text{Small}_5 \leftrightarrow \text{CFT}_4^3$

Stabilized compactifications $\leftrightarrow$ IR limit of concrete brane systems (w/ SUSY)

. Generalizations toward dS
AdS/CFT (and previously, BFSS matrix theory) formulate some backgrounds non-perturbatively, but did not (yet) get down to 4d:

BFSS: 
11d \leftrightarrow N \text{ 10-brane Q.M.}
4d (max susy) \leftrightarrow D7-branes on \mathbb{T}^7
\text{\emph{5 codim 2} } \rightarrow \log \text{ potential } \rightarrow N \leq 24

AdS/CFT: 
0 \quad \text{AdS}_2 \times S^2 \times \text{CY}
\text{want } L_{\text{AdS}} \rightarrow \infty \quad \text{small } \checkmark
\Rightarrow \text{IR divergences in AdS}_2
\[ \text{AdS/CFT } \oplus \text{ AdS}_4 \times \left\{ \begin{array}{c} S^3 \ (m) \\ S^3 / \mathbb{Z}_2 \\ \mathbb{CP}^3 \ (\text{IIA}) \end{array} \right\} \]

\[ \text{Internal } \times \text{ AdS} \]

No hierarchy of scales in Freund-Rubin compactifications.

Basic reason: In 11/10d Einstein equations \[ R_{MN} - \frac{1}{2} R g_{MN} = 8 \pi G T_{MN} \]

Internal + 4d flux

all three contributions are of the same order in the solution
Or in terms of the 4d effective potential:

\[ S = \int d^{10}x \sqrt{G} \left( \frac{R}{\alpha' g_s^2} + F_p^2 + \ldots \right) \]

4d potential energy

\[ U_R = -M_4^2 \frac{1}{R^2} + \ldots \]

\( R = \) curvature radius in string units.

\[ U_R \sim M_4^2 \Lambda \Rightarrow R_{AdS} \sim R \]

in Freund-Rubin
On the other hand, we can construct

\[ \text{(A)} \text{dS}_4 \times X_{\text{small}} \]

in an apparently large number of ways: \( \hat{B} \text{s} \) DRS BPMSS GKP + KKLT... suggesting a rich set of dual CFTs.

- Not a priori realized as near-horizon limit of brane system
- Can read off interesting properties:
  \[ N_{\text{d.o.f.}} \sim \frac{L^2}{\text{AdS}} M_p^2 \leq N \]
  \( b \sim \text{betti} \) #
  \( E \), \( B \), \( \text{d} \) #
  \( \text{AAB} \)
This talk:

SUSY Brane construction

Near Horizon

$AdS \times \text{Small}$

Using 7-branes to nearly cancel Curvature energy

$\Rightarrow$ Electric + Magnetic Flavors

Low energy: QFT

cf Aharony, Fayaztoush, Malacena
Strategy: Start from known, Freund-Rubin dual pair:

\[ \text{AdS}_5 \times S^1 \leftrightarrow \text{QFT} \]

(add at least brane construction)

add ingredients which cancel or nearly cancel \( U_p \)

stabilize the moduli \( \rightarrow \text{AdS}_{5/4} \)

\( \leftrightarrow \text{additional field content, couplings of QFT} \)

\( \leftrightarrow \text{CFT}_{4/3} \)
7-branes compete with curvature energy:

\[ U_7 = \frac{1}{g_s^2} R^{p} \cdot \frac{R^{q} \cdot R^2}{R^{6} \cdot \frac{1}{R^2}} \cdot M_4 \cdot \frac{1}{R^{2 q}} \]

\( \text{(p, q)} \)

Of course 7Bs, being Codimension 2, have large IR back reaction...
The interplay between curvature \& 7-brane energy is accurately captured using the techniques of F-theory: 

\[ T^2 \rightarrow X \]

\[ \downarrow \]

\[ B \]

\[ \gamma_{T^2} = C_0 + \frac{i}{g_s} \quad \text{in \ IIIB} \]
Plan: Start from Freund-Rubin dual pair:

\[ \text{AdS} \times S \leftrightarrow \text{QFT} \]

(add 7-branes, which cancel or nearly cancel \( U \))

(stabilize the moduli \( \to \text{AdS}_4 \))

(additional flavors \( \leftrightarrow \text{CFT}_3 \))

* in general, both electric and magnetic cf Douglas/Shenker, Argyres/Douglas, Argyres-Plesser-Seiberg-Witten
D3, D7 and Electric/Magnetic Matter

- 4d $N=2$ SU(2) SYM w/ $N_f$ hypermultiplets

Seiberg-Witten solution

\[ U \text{ (Coulomb branch)} \]

\( \otimes \) monopole

\( \otimes \) dyon

\( \otimes \) quark

AD/APS W: can change mass matrix $M$ such that mutually nonlocal matter is light.

- In brane constructions ($\text{Sen, Douglas, Seiberg}$)

\[ U \leftrightarrow \text{D3 position} \]

\[ \otimes \leftrightarrow \text{B position} \]

\( \gamma_{YM} \leftrightarrow \gamma_{PB} \)
The $T^3$ varying over $B$ can be described as

$$y^2 - x^3 - x f(u) z^4 - g(u) z^6 = 0$$

coordinates on $B$

i.e. as a degree 6 hypersurface in $WP^2(2,3,1)$.

For a Kähler base $B$, one can formulate the $T^3$ fibration $T^3 \rightarrow X \downarrow B$ as a hypersurface in $B \times WP^2(2,3,1)$, and as the target space of a $(2,2)$ gauged linear $\sigma$-model (GLSM) with $7$-branes line at the locus

$$\Delta = 27 g^2 + 4 f^3 = 0$$
Singualrities

• Some allowed (e.g., enhanced gauge symmetries)
• Some not allowed

A criterion for allowed singularities:

cf W, SW '90s

In the GLSM, singularities arise from extra non-compact branches of scalar field space

Compute (using GLSM)

\( \hat{C}_{\text{throat}} \)

If \( \hat{C}_{\text{throat}} \geq \hat{C}_{\text{bulk}} \)

then truly singular. (Otherwise linear dilaton → mass gap in throat)
This agrees with known cases...

\[
\int d^2 \theta \, P \left( y^2 - x^3 - x f(u) z^4 - g(u) z^6 \right)
\]

\((0\text{-term})\)

branch with \(\langle p \rangle \leq \langle z \rangle\); \(y = x = 0 = U\)

\[
\begin{align*}
\hat{C} &= 1 \\
\hat{C}_y &= 0 \\
\hat{C}_x &= \frac{1}{3} \\
\hat{C}_U &= 1 - \frac{2}{n}
\end{align*}
\]

\(\Rightarrow\) singular if \(n \geq 6\)

\[
\ldots \text{ and can be applied widely}
\]
let us start with IIB on
\[ Y_5 \times S^1 \] with \[ \int_Y F_5 = N_c \]
where \( Y_5 \) is an \( S^1 \) (Hopf) fibration over a Kähler base

\[ S^1 \to Y \]
\[ \downarrow \]
\[ B \]

\( \text{First study 7-branes on B} \)

\( 7\)-branes will be extended along fiber

\[ S^1 \to Y \]

Examples:
\[ Y = S^5, \quad B = \mathbb{C}P^2 \oplus \mathbb{R}P^2 \]
\[ Y = T^{1,1}, \quad B = \mathbb{C}P^1 \times \mathbb{C}P^1 \]
\[ Y = S^3, \quad B = \ldots \]
\[ Y = S^2 \times S^3, \quad B = \ldots \]

can have many 3-cycles

base of tonic \( C3 \) cones

" " non-tonic " "

Hanany ... Mortelli/Sparks/Yau
$S^1 \rightarrow Y \rightarrow B \rightarrow \text{site } R$ with "metric flux" $F_{\text{met}} = J_0$

Two classes of candidate examples:

1. 7-branes fully cancel curvature energy: e.g. 36 7-branes on $\mathbb{CP}^2$

$U_R + U_7 = 0 \quad (F\text{-theory on CY})$

 negative term from 0-planes

$R_{\text{AdS}} \gg R \gg R_f$
2. 7-branes nearly cancel $U^{(B)}_R$

$Y \times S^1 \times \text{AdS}_4$

$$U \sim M_p^4 \left( \frac{1}{R^4 R_6 R_8} \right) \left( \frac{R_f^2}{R^4} - \frac{\xi}{R^2} + \frac{N_c^2}{R^8 R_f^2} + \frac{Q_i^2}{R^2} \right)$$

$g_s \sim 1$

enforce with e.g. $E_n$ 7-branes $\Rightarrow$ stable minimum with $R_f \ll R \ll R_{\text{AdS}}$

Related (simpler) AdS$_5$ model:

$$U \big|_{g_s \sim 1} \sim M_5 \left( \frac{5}{(R^5 R_f)^{\frac{2}{3}}} \right) \left( \frac{R_f^2}{R^4} - \frac{\xi}{R^2} + \frac{N_c^2}{R^8 R_f^2} \right)$$
To get started, consider

$$Y = S^5 \text{ (topologically)}$$

Start from the $\mathbb{CP}^2$ model:

(2,2) chiral multiplets $U_1, U_2, U_3$

$U(1)$ Gauge symmetry

$$(U_1, U_2, U_3) \rightarrow e^{2\pi i \theta} (u_1, u_2, u_3)$$

$$D^2 = \left( |u_1|^2 + |u_2|^2 + |u_3|^2 - R^2 \right)^2$$

$D = 0$ alone gives $S^5$

$\gamma$ parameterizes $S^1_{\text{fiber}}$

$S^1_f \rightarrow S^5 \downarrow \mathbb{CP}^2$

$$ds^2_{S^5} = ds^2_{\mathbb{CP}^2} + R^2 (d\alpha + A)^2$$

$DA = J$

Gibbons, Pope
To add the 7-branes, want a $T^2$ fibration over $B = \mathbb{C}P^2$

Gauged Linear $\sigma$-model becomes

\[ u_1 \quad u_2 \quad u_3 \quad x \quad y \quad z \quad p \]

\[ T^2 \{ U(1) \}
\]

\[ x \]

\[ \mathbb{C}P^2 \{ U(1) \}
\]

\[ 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad q \neq 0 \]

\[ S_w = \int d^2 \sigma \ d^2 \theta \ P \left( y^2 - x^3 + f(u) x z^4 + g(u) z^6 \right) \]

Now, $\sum \limits_{\text{fields } I} q_I = 0$ is the Calabi-Yau condition (ensuring anomaly-free $U(1) \times U(1)$ R-symmetries appropriate to (2,2) SCFT) Witten
The running of $R^2$ in
$$D^2 = (|u_1|^2 + |u_2|^2 + |u_3|^2 + q_x |x|^2 + \frac{3}{2} q_x |y|^2 - |p|^2 - R^2)^2$$
is
$$M \frac{dR^2}{dM} = \sum_i q_i\text{.}$$

Now $\sum q_i = 3 + q_x$ and the degree of $G = y^2 - x^3 - f x t^4 - g t^6$
is $\deg G = \deg g = 6 q_x = 18 - 6 \sum q_i$

Fully canceling curvature energy means $\sum q_i = 0 \implies \deg g = 18$
$\implies \deg \Delta = 36 \implies 36$ 7-branes

(This agrees with naive result from $U_1 \to (\mathbb{R}^3 \times M) / \mathbb{R}$)

• The 7Bs are extended along $U(1)$ fiber
As next step, generalize to cases where we do not fully cancel the curvature energy. Consider $T^2$ fibration over $B = \mathbb{W}/\mathbb{P}^2$.

**Ranged Linear σ-model**

<table>
<thead>
<tr>
<th>$u_1$</th>
<th>$u_2$</th>
<th>$u_3$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$p$</th>
</tr>
</thead>
</table>

$T^2 \{ U(1) \}$  
$0 \quad 0 \quad 0 \quad 2 \quad 3 \quad 1 \quad 6$

$W^2 \{ U(1) \}$  
$W_1 \quad W_2 \quad W_3 \quad 0 \quad 0 \quad 0 \quad W_0 - \Sigma w_0$

$$S_w = \int d^2 \sigma \ d^2 \theta \ P \left( y^2 - x^3 - f(u) \times z^4 - g(u) z^6 \right)$$

Again $\beta_{R^2, \text{full}} \Sigma q = W_0$

in the full system including the $7B$s.
Again $\beta_{R^2,\text{full}} \sim \Sigma \rho = W_0$ 
in the full system including the 7Bs.

For $\text{WP}^2$ alone,

$$\beta_{R^2, \text{WP}^2} = W_1 + W_2 + W_3$$

$\Rightarrow$ if $W_0 < \ll W_1 + W_2 + W_3$

then we almost cancel the curvature.

$$U_{R^2, \text{full}} \sim M_{p} \left( \frac{g_5}{\text{Vol}} \right)^2 \text{Vol} \cdot \left( - \frac{\Sigma}{R^2} \right)$$

with $\Sigma \sim \frac{W_0}{W_1 + W_2 + W_3}$

(using the NLSM result $\beta \sim R\mu\nu$)
We can describe the full brane system (whose low-energy limit is the QFT) as F-theory on the following noncompact CY4:

<table>
<thead>
<tr>
<th>( u_0 )</th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( u_3 )</th>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>-6</td>
</tr>
</tbody>
</table>

\[ -W_0 = W_1 = W_2 = W_3 = 0, 0, W_0 - 3W_0 \]

\[ \leq W = 0 \text{ overall; cross-section geometry as above} \]

\[ \frac{u_j}{u_0} \text{ small if } W_0 < w_j : W_0 \sim \frac{1}{u_0}, W_j \sim \frac{1}{u_j} \]

- This + \( N_c \) D3-branes at tip \((U_0 = u_j = 0)\) is the brane construction. Altogether preserves 4 supercharges.
* Singularities of 7-branes:

\[ \text{Compute (using GLSM)} \]
\[ \hat{C}_{\text{throat}} : \hat{C} \]
\[ \text{If } \hat{C}_{\text{throat}} \geq \hat{C}_{\text{bulk}} \text{ then truly singular. (Otherwise linear dilaton } \rightarrow \text{ mass gap in throat)} \]

<table>
<thead>
<tr>
<th>( w_0 )</th>
<th>( W_1 )</th>
<th>( W_2 )</th>
<th>( W_3 )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1</td>
<td>( n )</td>
<td>( n )</td>
<td>-2n</td>
</tr>
</tbody>
</table>

\[ \rightarrow \text{monosingular but anisotropic} \]

<table>
<thead>
<tr>
<th>( w_0 )</th>
<th>( W_1 )</th>
<th>( W_2 )</th>
<th>( W_3 )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -w_0 )</td>
<td>( w - 5 )</td>
<td>( w )</td>
<td>( w + 5 )</td>
<td>( w_0 - 3w )</td>
</tr>
</tbody>
</table>

isotropic but singular \( g \sqrt{\sum u_1^2 u_2^2 - \frac{I}{18} u_3^2} \)

high degree \( \rightarrow \) high \( \hat{C} \)
So, choose nonsingular \( f \times g \)

that \( f \circ g \) is invariant.

Then \( w, \varnothing \) under which

the

conditions

\[
\begin{align*}
& \alpha_1 \cdot D - 3 \\
& \text{(extra \( UC \))}
\end{align*}
\]

restrict to isotropic

\[
\text{Subspace.}
\]

\[
\text{Term restricts to isotropic}
\]

Generalizations (isotropic, nonsingular)
Remarks

- 7-brane moduli approximately flat ($\beta = 0$ in sigma model)

- Sen (orientifold) limit may provide purely electric description

- Dasgupta/Mukhi constant coupling is enhanced symmetry point ($\Rightarrow$ extremum)
Number of degrees of freedom:

\[ \text{AdS}_5: \]

\[ N_{\text{d.o.f.}} \sim M_5^3 R_{\text{AdS}}^3 \sim \frac{1}{\varepsilon^3} N_{\text{d.o.f.}}^{(\text{no 7B}_5)} \]

\[ R_f \sim \varepsilon R_{\text{AdS}}, \quad R^2 \sim \varepsilon R_{\text{AdS}}^2 \]

\[ N_c \text{ D3}_5 \]

\[ \# \text{ of light states} \sim \frac{1}{(3^\frac{1}{4})^4} \cdot \frac{1}{3} \sim \frac{1}{3^3} \]
Future directions

- QFT content & couplings from brane system

- $dS_4$:

  - 0-planes
  - flux
  - slightly over-cancel curvature energy
  - Now no tachyons are allowed.

- GKP + KKLT use 7-branes (F-theory) and have 5-form flux. Interpret as above?

- Similar generalization of Gaiotto/Maldacena?